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ORIGINAL ARTICLE

Some Common Fixed Points in D-Metric Spaces

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Abstract:
Recently introduced a new topological structure of D-metric spaces and formulated the fundamental D-contraction principle is class of contraction mappings is further extended and generalized in various directions by several authors in the course of time and established several fixed point theorems or the single as well as pairs of commuting as well non commuting mappings in D-metric spaces.

INTRODUCTION

It is now clear that the fixed point theorems in D-metric spaces have some nice applications to approximation and optimization theory, and therefore, the field of D-metric fixed point theory is increasing with a good space.

In this paper, we proved fixed point theorems in metric spaces without any algebraic structure. We now consider spaces with a linear structure but non-linear mappings in them. In this paper we restrict our attention to normed spaces, but our main result will be extended to general locally convex spaces.

DEFINITION 1.1:

Let E be a vector space over. A mapping of E into R is called a norm on E if it satisfies the following axioms.

$$i)\ p(x) \geq 0 \, (x \in E)$$
$$ii)\ p(x) = 0 \textbf{ if and only if } x = 0$$
$$iii)\ p(x+y) \leq p(x) + p(y) \, (x, y \in E)$$

A vector space E with a specified norm on it called a normed space. The norm of an element x € E will usually be denoted by ||x||. A normed space is a metric space with the metric d(x,y) = x-y (x,yE)and the corresponding metric topology is called the normed topology. A normed linear space complete in the metric

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defined by the norm is called a Banach space. We now recall some definitions and well known properties of linear spaces. Two norms p1 and p2 on a vector space E are said to be equivalent if there exist positive constants k, k' such that

$$p_1(x) \leq k p_2(x),\ p_2(x) \leq k' p_1(x)(x \in E)$$

Two norms are equivalent if and only if they define the same topology.

DEFINITION 1.2:

A mapping f of a vector space E into R is called a linear functional on E if it satisfies

$$i)\ f(x+y) = f(x) + f(y)(x, y \in E)$$
$$ii)\ f(\alpha x) = \alpha f(x)(x \in E, \alpha \in R).$$

A mapping p : E → R is called a sub linear functional if

$$i)\ p(x+y) \leq p(x) + p(y)(x, y \in E)$$
$$ii)\ p(\alpha x) = \alpha p(x)(x \in E, \alpha \geq 0).$$

HAHN – BANACH THEOREM:

Let E_o be a subspace of a vector space E over R; let p be a sub linear functional on E and let f_o be a linear functional on eo that satisfies.

$$f_0(x) \leq p(x)(x \in E_0).$$

Then there exists a linear functional f on E that satisfies.

$$i)\ f(x) \leq p(x)(x \in E)$$
$$ii)\ f(x) = f_0(x)(x \in E_0).$$

For the proof refer to Dunford and Schwartz or Day.

COROLLARY:

Given a sub linear functional on E and x_0 € E, there exists a linear functional f such that

$$f(x_0) = p(x_0), f(x) \leq p(x)(x \in E).$$

In particular, a norm being a sub linear functional. Giben a point x0 of a normed space E, there exists a linear functional f on E such that.

$$|f(x)| \leq \|x\| (x \in E) and\ f(x_0) = \|x_0\|$$

DEFINITION 1.3:

A norm p on a vector space E said to be strictly convex if p (x + y) = p(x)+ p(y) only when x and y are linearly dependent.

THEOREM (CLARKSON) 1.1:

If a normed space E has a countable every-where dense subset, then there exists a strictly convex norm on E equivalent to the given norm.

PROOF:

Let S denote the surface of the unit ball in E,

$$S = \{x : \|x\| = 1\}$$

Then there exists a countable se (xn) of points of S that is dense in S. for each n, there exists a linear functional fn on E such that

$$f_n(x_n) = \|x_n\| = 1 \textbf{ and } |f_n(x)| \le \|x\| \, (x \in E)$$

If x ‡ 0, then fn(x) ‡ 0 for some n. For by homogeneity, it is enough to consider x with $\|x\| = 1$, and for such x there exists n with $\|x - x_n\| < \frac{1}{2}$. But then

$$f_n(x) = f_n(x_n) + f_n(x - x_n) \ge 1 - f_n(x - x_n)$$

$$\ge 1 - \|x - x_n\| > \frac{1}{2}$$

We now take $p(x) = \|x\| + \left\{\sum_{n=1}^{\infty} 2^{-n}(f_n(x))^2\right\}^{\frac{1}{2}}$. It is easily verifies that p is a norm on E and that

$$\|x\| \le p(x) \le 2\|x\|$$

Finally p is strictly convex. To see this, suppose that

$$p(z+y) = p(x) + p(y).$$

and write $\xi_n = f_n(x), y_n = f_n(y)$. Then

$$\left\{\sum_{n=1}^{\infty} 2^{-n}(\xi_n + \eta_n)^2\right\}^{\frac{1}{2}} = \left\{\sum_{n=1}^{\infty} s^{-n}\xi_n^2\right\}^{\frac{1}{2}} = \left\{\sum_{n=1}^{\infty} 2^{-n}\eta_n^2\right\}^{\frac{1}{2}}$$

And we have the case of equality in Minkowsiki's inequality. It follows that the sequence (ξ_n) and (η_n) are linearly dependent. Thus there exist ,v. not both zero, such that

$$\lambda\xi_n + \mu\eta_n = 0 \, (n = 1, 2, \ldots)$$

But this implies that

$$f_n(\lambda x + \mu y) = 0 \, (n = 1, 2, \ldots)$$

and so x + y = 0. This completes the proof.

LEMMA 1.1:

Let K be a compact convex subset of a normed space E with a strictly convex norm. Then to each point x of E corresponds a unique point px of K at K at minimum distance from x, i.e., with.

$$\|x-Px\| = \inf \{\|x-y\| : y\varepsilon K\}$$

And the mapping x $\rightarrow$ Px is continuous in E.

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