Golden Research Thoughts
Volume 2, Issue. 3, Sept 2012
ISSN:-2231-5063

Available online at www.aygrt.net

ORIGINAL ARTICLE

"Mass – Spring Damper System" As An Application Of Laplace Transformation

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ABSTRACT:

The mechanical system "Mass spring damper system" is use to modelled many real world system. The dynamic pattern of the system leads second order linear ordinary differential equation. The stiffness, damping and mass involve in the calculation of second order differential equation. Here we solve the second order differential equation by Laplace transformation.

Keywords:

Amplitude; oscillation, damper; Laplace Transform; Inverse Laplace transform

1. INTRODUCTION

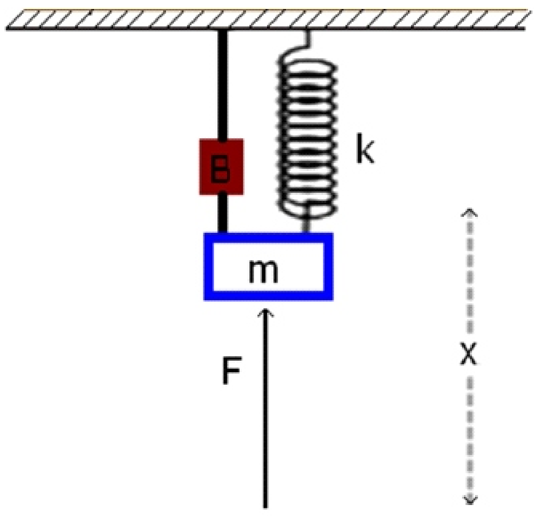
A Laplace transform is an extension of the continuous time Fourier transform motivated by the fact that this transform can be used to a wider class of signals than Fourier transform can [2]. Laplace transform provides the solution of differential equations with given boundary condition without finding the general solution. Laplace transformation is used for solving linear differential equations, integral equations, and partial differential equations.[2] The Laplace transformation is a very powerful integral transformation used to switch a function from the time domain to the s-domain. There are many applications of Laplace transformation, here we represent one of the applications of Laplace transformations," Mass –spring damping system".

To solve linear ordinary differential equation by Laplace transform method ,we first convert the equation in the unknown function f(t) into an equation in F(s) and find F(s).For that we use the following result,

$$L\{f'(t)\} = sF(s) - f(s)$$
$$L\{f''(t)\} = s^2 F(s) - sf(0) - f'(0)$$

2. MASS-SPRING DAMPER SYSTEM

Damping is an effect that reduces the amplitude of oscillation in an oscillatory system.[1] A mass attached to the spring and damper shown in figure. Let m be a mass and spring is rigidly supported from one end.



In this system x represent the position.
If x=0 then it is rest position,
If x>0 then downwards displacement,
If x<0 then upward displacement.
(1)k>0 is spring constant
(2)A_d>0 is damping constant
(3)A_d

2.1 EQUATION OF MOTION

By Newton's second law of motion, the sum of forces acting on m equals m$\frac{d^2x}{dt^2}$

$$m\frac{d^2x}{dt^2} = -kx - A_d\frac{dx}{dt} + f(t)$$

So, the equation of motion is,

$$m\frac{d^2x}{dt^2} + A_d\frac{dx}{dt} + kx = f(t)$$

his is differential equation which occurs in harmonic oscillator.
(1) If Ad =0 ,the motion is undamped,
(2) If Ad ≠0, the motion is damped.
(3) If f(t) =0 ,there is no impressed forces then the motion is called unforced. Otherwise the motion is forced.
We can also write equation of motion as,

$$\frac{d^2x}{dt^2}+\frac{\Lambda_d}{m}\frac{dx}{dt}+\frac{k}{m}x=\frac{f(t)}{m}$$

2.2 Solution of equation of motion

Let we consider x (0) =0, x'(0) =0.
Taking Laplace transform on both side,
we get
$\{s^2X(s)-sx(0)-x'(0)\}+\frac{\Lambda_d}{m}\{sX(s)-x(0)\}+\frac{k}{m}X(s)=L(\frac{f(t)}{m})$
Substitute x(0)=x'(0)=0, we get

$(s^2+\frac{\Lambda_d}{m}s+\frac{k}{m})$ X(s) = $L(\frac{f(t)}{m})$
So, $X(s)=\frac{1}{(s^2+\frac{\Lambda_d}{m}s+\frac{k}{m})}L(\frac{f(t)}{m})$ __________ (1)

Case1:

If $\frac{\Lambda_d^2}{4m^2}<\frac{k}{m}$ then

$$X(s)=\frac{1}{\left(s^2+\frac{\Lambda_d}{m}s+\frac{\Lambda_d^2}{4m^2}+\frac{k}{m}-\frac{\Lambda_d^2}{4m^2}\right)}L(\frac{f(t)}{m})$$

$$X(s)=\frac{1}{\left(s+\frac{\Lambda_d}{2m}\right)^2+\left(\frac{k}{m}-\frac{\Lambda_d^2}{4m^2}\right)}$$

Now take inverse Laplace transform,

$$x(t)=e^{-\frac{\Lambda_d}{2m}t}\left(\frac{1}{\sqrt{\frac{k}{m}-\frac{\Lambda_d^2}{4m^2}}}\sin\sqrt{\frac{k}{m}-\frac{\Lambda_d^2}{4m^2}}\;t\right)$$

Case 2:

If $\frac{\Lambda_d^2}{4m^2}>\frac{k}{m}$ then

$$X(s)=\frac{1}{\left(s^2+\frac{\Lambda_d}{m}s+\frac{\Lambda_d^2}{4m^2}+\frac{k}{m}-\frac{\Lambda_d^2}{4m^2}\right)}L(\frac{f(t)}{m})$$

$$X(s)=\frac{1}{\left(s+\frac{\Lambda_d}{2m}\right)^2-\left(\frac{\Lambda_d^2}{4m^2}-\frac{k}{m}\right)}$$

Now take inverse Laplace transform,

$$x(t)=e^{-\frac{\Lambda_d}{2m}t}\left(\frac{1}{\sqrt{\frac{\Lambda_d^2}{4m^2}-\frac{k}{m}}}\sinh\sqrt{\frac{\Lambda_d^2}{4m^2}-\frac{k}{m}}\;t\right)$$

Case 3:

If $\frac{\Lambda_d^2}{4m^2}=\frac{k}{m}$ then

$$X(s)=\frac{1}{\left(s^2+\frac{\Lambda_d}{m}s+\frac{\Lambda_d^2}{4m^2}\right)}L(\frac{f(t)}{m})$$

$$X(s)=\frac{1}{\left(s+\frac{\Lambda_d}{2m}\right)^2}$$

Now take inverse Laplace transform,

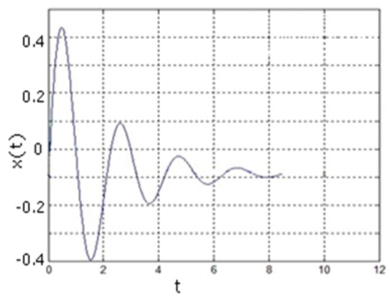
$$x(t)=e^{-\frac{\Lambda_d}{2m}t}$$

3. Classification of system.

When the values of k, Λ_d, and m are change the oscillation is change.
If we take k as constant and change the values of 'Λ_d' and 'm' then oscillation of the system is change. So for one spring we can find different kind of systems. From different oscillation Mass spring damper system is classified into following categories.

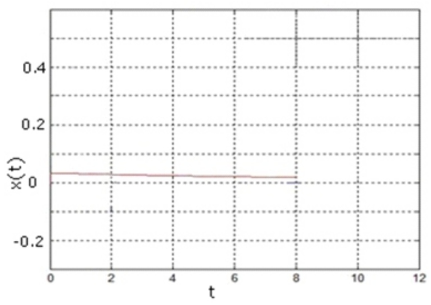
3.1 Under damped system ($\frac{\Lambda_d{}^2}{4m^2} < \frac{k}{m}$)

In case 1 the solution is clearly damped oscillation. The system oscillates with the amplitude gradually decreasing to zero.[1] The graph of $x(t) \to t$ is shown in following figure.



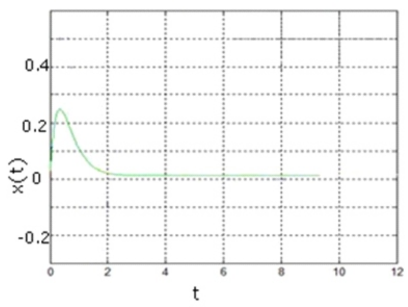
3.2 Over damped system ($\frac{\Lambda_d{}^2}{4m^2} > \frac{k}{m}$)

Case 2 is the over damped condition. The system returns to equilibrium without oscillating. [1]
The graph of $x(t) \to t$ is shown in following figure



3.3 Critical damped system ($\frac{\Lambda_d{}^2}{4m^2} = \frac{k}{m}$)

Case 3 is the critical damped condition. The system returns to equilibrium as quickly as possible without oscillating.[1]
The graph of $x(t) \to t$ is shown in following figure



3.3 Undamped system (a=0)

If the damper constant 'a' is zero then system is undamped. When there is no any friction the system oscillated with constant amplitude such oscillation is known as undamped oscillation. The system oscillates as its natural resonant frequency. [1]

Here A_d=0, therefore

$$x(t) = e^{-\frac{0}{2m}t}\left(\frac{1}{\sqrt{\frac{k}{m}-\frac{0^2}{4m^2}}}\sin\sqrt{\frac{k}{m}-\frac{0^2}{4m^2}}\;t\right)$$

$$x(t) = \left(\frac{1}{\sqrt{\frac{k}{m}}}\sin\sqrt{\frac{k}{m}}\;t\right)$$

The graph of $x(t) \rightarrow t$ is shown in following figure

4 Conclusions

The vibration created by the mass spring damper system is depend on the damping constant 'A_d' and spring constant 'k'.By using the different values of 'A_d' and 'k', we will get the various mechanical system. By equation of the x(t), we can find the accurate damping constant and the spring constant for given mass. We can also get the resonance and the beats occurrences.

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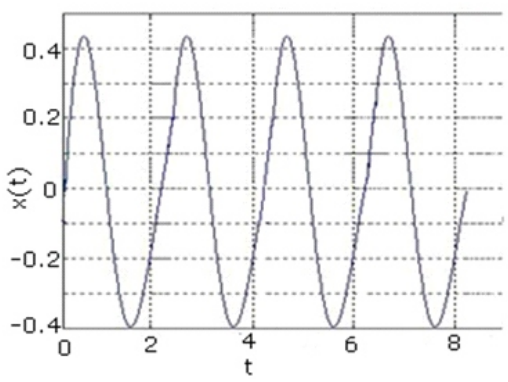

When frequency of the vibration is same as the natural frequency of the system resonance occurs. If the frequency of the vibration is greater than the natural frequency of the system than system can be destructed. When the lighted damping system is forced by vibration whose frequency is closed to the natural frequency than beats occurs.