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TOPOLOGY SEPARATION PROPERTIES

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Abstract:

In this paper some separation properties using -open sets in topological spaces are defined and their relationships with some other properties are studied.

AMS Classification: 54 D 10, 54 G20

KEYWORDS :

α - C_0 , α - C_1 , weakly α - C_0 , and weakly α - C_1

INTRODUCTION :

Throughout this paper by a space X we mean it is a topological space. If A is any subset of a space X , then $cl(A)$, $int(A)$ and $C(A)$ denote the closure, the interior and the complement of A respectively. A subset A of a space X is called semi-open [6] (resp. α -open [11], pre-open [10]) if $A \subseteq cl(int(A))$ (resp. $A \subseteq int(cl(int(A)))$, $A \subseteq int(cl(A))$). The class of all semi-open (resp. pre-open, α -open) subsets of a space X is denoted by $SO(X)$ (resp. $PO(X)$, $\alpha(X)$). The complement of semi-open (resp. pre-open, α -open) subset of a space is called semi-closed (resp. pre-closed, α -closed) set. $scl(A)$ (resp. $pcl(A)$, $\alpha cl(A)$) denote the semi-closure (resp. pre-closure, α -closure) of the set A . Maheswari and Tap [9] called a subset B of a space X as feebly open if there is an open set G such that $G \subseteq B \subseteq scl(G)$. Later Janković and Reilly [5] observed that feebly open sets are precisely α -open sets.

2. PREREQUISITES

Let us recall the following definitions:

DEFINITION 2.1. A space (X, τ) is called

- (1) C_0 (semi- C_0) if, for $x, y \in X$, $x \neq y$, there exists $G \in \tau$ ($SO(X)$) such that $cl(G) \setminus (scl(G))$ contains only one of x and y but not the other;
- (2) C_1 (semi- C_1) if, for $x, y \in X$, $x \neq y$, there exist $G, H \in \tau$ ($SO(X)$) such that $x \in cl(G) \setminus (scl(G))$, $y \in cl(H) \setminus (scl(H))$ but $x \notin cl(H) \setminus (scl(H))$ and $y \notin cl(G) \setminus (scl(G))$;
- (3) w - C_0 [3] if $\bigcap \ker(x) = \emptyset$, where $\ker(x) = \bigcap \{G : x \in G \in \tau\}$; $x \in X$
- (4) weakly semi- C_0 if $\bigcap \ker(x) = \emptyset$, where $\ker(x) = \bigcap \{G : x \in G \in SO(X)\}$; $x \in X$
- (5) R_0 [2] if $cl(\{x\}) \subseteq G$ whenever $x \in G \in \tau$;

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TOPOLOGY SEPARATION PROPERTIES



- (6) semi- R_0 [8] if, for $x \in G \in SO(X)$, $scl(\{x\}) \subseteq G$;
- (7) weakly R_0 [3] if $\bigcap_{x \in X} cl(\{x\}) = \phi$;
- (8) weakly semi- R_0 [1] if $\bigcap_{x \in X} scl(\{x\}) = \phi$;
- (9) weakly pre- R_0 if $\bigcap_{x \in X} pcl(\{x\}) = \phi$;
- (10) weakly pre- C_0 if $\bigcap_{x \in X} pker(x) = \phi$, where $pker(x) = \bigcap \{G : x \in G \in PO(X)\}$;
- (11) α -space [4] if every α -open set in it is open.

Maheswari and Prasad [7] introduced semi- T_i ($i = 0, 1, 2$) axiom, which is weaker than T_i ($i = 0, 1, 2$) axiom.

We use the following sets and classes for counter examples.

Let $X = \{a, b, c, d\}$, $Y = \{a, b, c\}$, $Z = \{a, b, c, d, e, f\}$

Let $\tau_1 = \{\phi, X, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$, $\sigma_1 = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$,

$\eta_1 = \{\phi, Z, \{a, c, e\}, \{b, d, f\}\}$, $\sigma_2 = \{\phi, Y, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$,

$\tau_2 = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$,

$\sigma = \{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$.

THEOREM 2.2.

1. Every C_1 (semi- C_1) space is a C_0 (semi- C_0).
2. Every C_0 (C_1) space is a semi- C_0 (semi- C_1).
3. Every R_0 space is a weakly R_0 [3].
4. Every weakly R_0 space is weakly semi- R_0 [1].

Every semi- C_0 (semi- C_1) space is semi- T_0 (semi- T_1).

PROOF. Omitted.

REMARK 2.3. (X, σ_1) is semi- C_0 but not C_0 . (X, τ_2) is semi- C_1 , C_0 but not C_1 . (Y, σ_2) is semi- T_0 but not semi- C_0 . (Z, η_1) is an α -space but not an α - C_0 .

3. α - C_0 , α - C_1 , weakly α - C_0 and weakly α - R_0 spaces

Now we introduce the following separation properties using α -open sets in spaces.

DEFINITION 3.1. A space X is called

- (1) α - C_0 if, for $x, y \in X, x \neq y$, there exists $G \in \alpha(X)$ such that $\alpha\text{cl}(G)$ contains only one of x and y but not the other;
- (2) α - C_1 if, for $x, y \in X, x \neq y$, there exist $G, H \in \alpha(X)$ such that $x \in \alpha\text{cl}(G), y \in \alpha\text{cl}(H)$ but $x \notin \alpha\text{cl}(H)$ and $y \notin \alpha\text{cl}(G)$;
- (3) weakly α - C_0 if $\bigcap_{x \in X} \alpha\text{ker}(x) = \emptyset$, where $\alpha\text{ker}(x) = \bigcap \{G : x \in G \in \alpha(X)\}; x \in X$
- (4) weakly α - R_0 if $\bigcap_{x \in X} \alpha\text{cl}(\{x\}) = \emptyset; x \in X$

THEOREM 3.2.

1. Every α - C_1 space is α - C_0 .
2. Every α - C_0 (α - C_1) space is semi- C_0 (semi- C_1).
3. Every weakly α - R_0 space is weakly semi- R_0 and weakly pre- R_0 .
4. Every w- C_0 space is weakly α - C_0 .
5. Every weakly α - C_0 space is weakly semi- C_0 and weakly pre- C_0 .
6. Every α - C_0 (α - C_1) space is semi- T_0 (semi- T_1).
7. Every weakly R_0 space is weakly α - R_0 .
8. Weakly α - R_0 ness and weakly α - C_0 ness are independent notions.

REMARK 3.3.

(X, τ_2) is α - C_0 but not α - C_1 . (X, σ_1) is semi- C_0 , semi- C_1 , but neither α - C_1 nor α - C_0 . (Y, σ) is weakly semi- R_0 but not weakly α - R_0 . (X, τ_2) is weakly α - C_0 but not weakly α - R_0 . (X, τ_1) is weakly α - R_0 but not weakly α - C_0 .

THEOREM 3.4.

A space X is weakly α - R_0 if and only if $\alpha\text{ker}(x) \neq X$ for each $x \in X$.

PROOF:

Necessity – If there is some $x_0 \in X$ with $\alpha\text{ker}(x_0) = X$, then X is the only α -open set containing x_0 . This implies that $x_0 \in \alpha\text{cl}(\{x\})$ for every $x \in X$. Hence $\bigcap_{x \in X} \alpha\text{cl}(\{x\}) \neq \emptyset$, a contradiction.

Sufficiency – If X is not weakly α - R_0 , then choose some $x_0 \in \bigcap_{x \in X} \alpha\text{ker}(x)$.

This implies that every α -neighborhood of x_0 contains every point of X . Hence

$\alpha\text{ker}(x_0) = X$.

THEOREM 3.5.

A space X is weakly α - C_0 if and only if for each $x \in X$, there exists a proper α -closed set containing x_0 .

PROOF:

Necessity – Suppose there is some $x_0 \in X$ such that X is the only α -closed set containing x_0 . Let U be any proper α -open subset of X containing a point x . This implies that $C(U) \neq X$. Since $C(U)$ is α -closed, we have $x_0 \in C(U)$. So $x_0 \in U$. Thus

$x_0 \in \bigcap_{x \in X} \ker(x)$ for any point x of X , a contradiction.

Sufficiency – If X is not weakly α - C_0 , then choose $x_0 \in \bigcap_{x \in X} \alpha \ker(x)$. So x_0 belongs to $\alpha \ker(x)$ for any $x \in X$. This implies that X is the only α -open set, which contains the point x_0 , a contradiction.

THEOREM 3.6.

Every α - C_0 (α - C_1) space is weakly α - C_0 .

PROOF:

If $x, y \in X$ such that $x \neq y$, where X is an α - C_0 space, then without loss of generality, we can assume that there exists $U \in \alpha(X)$ such that $x \in \alpha \text{cl}(U)$ but $y \notin \alpha \text{cl}(U)$. This implies that $U \neq \emptyset$. Hence we can choose z in U . Now $\alpha \ker(z) \cap \alpha \ker(y)$

$\subseteq U \cap (\alpha \text{cl}(U)) \subseteq (\alpha \text{cl}(U)) \cap C(\alpha \text{cl}(U)) = \emptyset$.

Hence $\bigcap_{x \in X} \ker(x) = \emptyset$. Since every α - C_1 space is also α - C_0 , it is also clear that every α - C_1 space is weakly α - C_0 .

REMARK 3.7.

The converse of the above theorem need not be true since

(Z, η_1) is weakly α - C_0 but not α - C_0 . The space (Y, σ) is both α - C_0 and weakly α - C_0 but not α - C_1 .

THEOREM 3.8.

The property of being an α - C_0 space is not hereditary.

PROOF :

Consider the space (Y, σ_1) . Let $S = \{a, c\}$ and σ_1^* be the relative topology on S . It is easy to verify that (Y, σ_1) is α - C_0 but it's subspace (S, σ_1^*) is not α - C_0 .

CONCLUSION :

We have shown that some separation properties using α -open sets in topological spaces .

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