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ORIGINAL ARTICLE

TOPOLOGY SEPARATION PROPERTIES

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Abstract:

In this paper some separation properties using -open sets in topological spaces are defined and their relationships with some other properties are studied.

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KEYWORDS :

α-C_0, α-C_1, weakly α-C_0, and weakly α-C_1

INTRODUCTION :

Throughout this paper by a space X we mean it is a topological space. If A is any subset of a space X, then cl(A), int(A) and C(A) denote the closure, the interior and the complement of A respectively. A subset A of a space X is called semi-open [**6**] (resp. α-open [**11**], pre-open [**10**]) if $A \subseteq cl(int(A))$ (resp. $A \subseteq int(cl(int(A)))$, $A \subseteq int(cl(A))$). The class of all semi-open (resp. pre-open, α-open) subsets of a space X is denoted by SO(X) (resp. PO(X), α(X)). The complement of semi-open (resp. pre-open, α-open) subset of a space is called semi-closed (resp. pre-closed, α-closed) set.scl(A) (resp. pcl(A), αcl(A)) denote the semi-closure (resp. pre-closure, α-closure) of the set A. Maheswari and Tap [**9**] called a subset B of a space X as feebly open if there is an open set G such that $G \subseteq B \subseteq scl(G)$. Later Janković and Reilly [**5**] observed that feebly open sets are precisely α-open sets.

2. PREREQUISITES

Let us recall the following definitions:

DEFINITION 2.1. A space (X, τ) is called

(1) C_0 (semi-C_0) if, for x, y $\in$ X, x $\neq$ y, there exists G $\in$ τ (SO(X)) such that cl(G) (scl(G)) contains only one of x and y but not the other;

(2) C_1 (semi-C_1) if, for x, y $\in$ X, x $\neq$ y, there exist G, H $\in$ τ (SO(X)) such that x $\in$ cl(G) (scl(G)), y $\in$ cl(H) (scl(H)) but x $\notin$ cl(H) (scl(H)) and y $\notin$ cl(G) (scl(G));

(3) w-C_0 [**3**] if $\cap\, ker(x) = \phi$, where $ker(x) = \cap \{G: x \in G \in \tau\}$; $x \in X$

(4) weakly semi-C_0 if $\cap\, sker(x) = \phi$, where $sker(x) = \cap \{G: x \in G \in SO(X)\}$; $x \in X$

(5) R_0 [**2**] if $cl(\{x\}) \subseteq G$ whenever $x \in G \in \tau$;

(6) semi-R_0 [**8**] if, for $x \in G \in SO(X)$, $scl(\{x\}) \subseteq G$;

(7) weakly R_0 [**3**] if $\bigcap_{x \in X} cl(\{x\}) = \phi$;

(8) weakly semi-R_0 [**1**] if $\bigcap_{x \in X} scl(\{x\}) = \phi$;

(9) weakly pre-R_0 if $\cap\ pcl(\{x\}) = \phi$; $x \in X$

(10) weakly pre-C_0 if $\cap\ pker(x) = \phi$, where $pker(x) = \cap \{G: x \in G \in PO(X)\}$; $x \in X$

(11) α-space [**4**] if every α-open set in it is open.

Maheswari and Prasad [**7**] introduced semi-T_i (i = 0, 1, 2) axiom, which is weaker than T_i (i = 0, 1, 2) axiom.

We use the following sets and classes for counter examples.

Let X = {a, b, c, d}, Y = {a, b, c}, Z = {a, b, c, d, e, f}

Let τ_1 = **{**ϕ, X, {b}, {a, b}, {b, c}, {a, b, c}**}**, σ_1 = **{**ϕ, X, {a}, {b}, {a, b}**}**,

η_1 = **{**ϕ, Z, {a, c, e}, {b, d, f}**}**, σ_2 = **{**ϕ, Y, {a}, {b}, {a, b}, {b, c}**}**,

τ_2 = **{**ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}**}**,

σ = **{**ϕ, Y, {a}, {b}, {a, b}**}**.

THEOREM 2.2.

1. Every C_1 (semi-C_1) space is a C_0 (semi-C_0).
2. Every C_0 (C_1) space is a semi-C_0 (semi-C_1).
3. Every R_0 space is a weakly R_0 [**3**].
4. Every weakly R_0 space is weakly semi-R_0 [**1**].

Every semi-C_0 (semi-C_1) space is semi-T_0 (semi-T_1).

PROOF. Omitted.

REMARK 2.3. (X, σ_1) is semi-C_0 but not C_0. (X, τ_2) is semi-C_1, C_0 but not C_1. (Y, σ_2) is semi-T_0 but not semi-C_0. (Z, η_1) is an α-space but not an α-C_0.

3. α-C_0, α-C_1, weakly α-C_0 and weakly α-R_0 spaces

Now we introduce the following separation properties using α-open sets in spaces.

DEFINITION 3.1. A space X is called

(1) α-C_0 if, for x, y $\in$ X, x $\neq$ y, there exists G $\in$ α(X) such that αcl(G) contains only one of x and y but not the other;

(2) α-C_1 if, for x, y $\in$ X, x $\neq$ y, there exist G, H $\in$ α(X) such that x $\in$ αcl(G), y $\in$ αcl(H) but x $\notin$ αcl(H) and y $\notin$ αcl(G);

(3) weakly α-C_0 if $\bigcap_{x\in X} \alpha\ker(x) = \phi$, where $\alpha\ker(x) = \cap \{G: x \in G \in \alpha(X)\}$;

(4) weakly α-R_0 if $\bigcap_{x\in X} \alpha cl(\{x\}) = \phi$;

THEOREM 3.2.

1. Every α-C_1 space is α-C_0.
2. Every α-C_0 (α-C_1) space is semi-C_0 (semi-C_1).
3. Every weakly α-R_0 space is weakly semi-R_0 and weakly pre-R_0.
4. Every w-C_0 space is weakly α-C_0.
5. Every weakly α-C_0 space is weakly semi-C_0 and weakly pre-C_0.
6. Every α-C_0 (α-C_1) space is semi-T_0 (semi-T_1).
7. Every weakly R_0 space is weakly α-R_0.
8. Weakly α-R_0ness and weakly α-C_0ness are independent notions.

REMARK 3.3.

(X, τ_2) is α-C_0 but not α-C_1. (X, σ_1) is semi-C_0, semi-C_1, but

neither α-C_1 nor α-C_0. (Y, σ) is weakly semi-R_0 but not weakly α-R_0. (X, τ_2) is

weakly α-C_0 but not weakly α-R_0. (X, τ_1) is weakly α-R_0 but not weakly α-C_0.

THEOREM 3.4.

A space X is weakly α-R_0 if and only if $\alpha\ker(x) \neq X$ for each x $\in$ X.

PROOF:

Necessity – If there is some $x_0 \in X$ with $\alpha\ker(x_0)) = X$, then X is the only α- open set containing x_0. This implies that $x_0 \in \alpha cl(\{x\})$ for every x $\in$ X. Hence $\bigcap_{x\in X} \alpha cl(\{x\}) \neq \phi$, a contradiction.

Sufficiency – If X is not weakly α-R_0, then choose some $x_0 \in \bigcap_{x\in X} \alpha\ker(x)$.

This implies that every α-neighborhood of x_0 contains every point of X. Hence

$\alpha\ker(x_0) = X$.

THEOREM 3.5.

A space X is weakly α-C_0 if and only if for each $x \in X$, there exists a proper α-closed set containing x_0.

PROOF:

Necessity – Suppose there is some $x_0 \in X$ such that X is the only α-closed set containing x_0. Let U be any proper α-open subset of X containing a point x. This implies that $C(U) \neq X$. Since C(U) is α-closed, we have $x_0 \in C(U)$. So $x_0 \in U$. Thus

$x_0 \in \bigcap_{x \in X} \ker(x)$ for any point x of X, a contradiction.

Sufficiency – If X is not weakly α-C_0, then choose $x_0 \in \bigcap_{x \in X} \alpha\ker(x)$. So x_0 belongs to $\alpha\ker(x)$ for any $x \in X$. This implies that X is the only α-open set, which contains the point x_0, a contradiction.

THEOREM 3.6.

Every α-C_0 (α-C_1) space is weakly α-C_0.

PROOF:

If x, $y \in X$ such that $x \neq y$, where X is an α-C_0 space, then without loss of generality, we can assume that there exists $U \in \alpha(X)$ such that $x \in \alpha cl(U)$ but $y \notin \alpha cl(U)$. This implies that $U \neq \phi$. Hence we can choose z in U. Now $\alpha\ker(z) \cap \alpha\ker(y)$

$\subseteq U \cap (\alpha cl(U)) \subseteq (\alpha cl(U)) \cap C(\alpha cl(U)) = \phi$.

Hence $\bigcap_{x \in X} \ker(x) = \phi$. Since every α-C_1 space is also α-C_0, it is also clear that every α-C_1 space is weakly α-C_0.

REMARK 3.7.

The converse of the above theorem need not be true since

(Z, η_1) is weakly α-C_0 but not α-C_0. The space (Y, σ) is both α-C_0 and weakly α-C_0

but not α-C_1.

THEOREM 3.8.

The property of being an α-C_0 space is not hereditary.

PROOF :

Consider the space (Y, σ_1). Let S = {a, c} and σ_1^* be the relative topology on S. It is easy to verify that (Y, σ_1) is α-C_0 but it's subspace (S, σ_1^*) is not α-C_0.

CONCLUSION :

We have shown that some separation properties using -open sets in topological spaces .

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