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ORIGINAL ARTICLE

A COMMON FIXED POINT THEOREM IN A REAL VALUED FUNCTIONS DEFINED ON INTERVALS

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ABSTRACT

In this paper we obtain a common fixed point theorem only the real valued function defined on intervals. We try to develop the differentiable function. In this paper we see that every rational function is differentiable except at the point where the denominator is zero

INTRODUCTION

Here we discuss the detail analysis of real valued differentiable function. Where f is considered as real valued function defined on an interval I R .

The concept of derivative of a function and method of obtaining the derivatives of some functions has been solve by different lemma's & example of following

KEY WORDS

differentiable function , continuous functions , limit , real valued function , neighborhood

DEFINITION 1

As F is said to have a derivatives at x = a if ——— exist. If the limit exists, then f is said to be differentiable at a and its derivative is denoted by

———

Hence if E is the set of points of I at which $f'(a)$ exists and E $=\emptyset$. Then f' is itself a real valued function on E. If f' is defined at every point of E , then f is said to be differentiable on E. It is possible that E $=\emptyset$ and there are functions which are differentiable at some points in the domain but not at other points of the domain.

LEMMA 1

If the real valued function f is differentiable at the point a ∈ R ,

then f is continuous at x = a

PROOF :-

Let a function f be differentiable at x = a

Then $\lim_{h\to 0}\frac{\mathrm{f(x+h)-f(x)}}{h} = f'(x)$ exists.

In order to show that f is continuous at x = a we have to show that

$$\lim_{x\to a} f(x) = f(a)$$

i.e
$$\lim_{x\to a} [\, f(x) - f(a)\,] = 0$$

$$= f'(x) \times 0 = 0$$

The converse of the above statement is 'even if the function f is continous at x = a it need not be differentiable at x = a'

EXAMPLE 1:-

The f (x) = x|(x)| for x ∈ R than $f'(x) = 2\,|x|$ for every x in R from the definition of the function $f(x) = x^2$ if $\mathrm{x} > 0$ and $f(x) = -x^2$ for $\mathrm{x} < 0$. If $\mathrm{x} > 0$

Ans :

we have,

$$f'(x) = \lim_{\mathrm{h}\to 0}\frac{\mathrm{f(x+h)-f(x)}}{\mathrm{h}}$$

$$= \lim_{\mathrm{h}\to 0}\frac{\mathrm{(x+h)^2 - x^2}}{\mathrm{h}}$$

Since a+ h $>$ 0 when $|h|$ is sufficiently small

$$f'(x) = \lim_{h\to 0} \frac{2xh+h^2}{h}$$

$$= \lim h_{h\to 0}(2x + h)$$

$$= 2x$$

If x$<$ 0 then

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{-(x+h)^2 - x^2}{h}$$

Since x+h $<$ 0 , where $|h|$ is sufficiently small ,

$$f'(x) = \lim_{h\to 0} \frac{-2xh-h^2}{h}$$

$$= \lim_{h\to 0}(-2x - h)$$

$$= -2x$$

Let us consider the case when x = 0

$$f'(0) = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h}$$

$$= \lim_{h\to 0} \frac{h|h|}{h}$$

$$= \lim_{h\to 0}|h| = 0$$

Combining all the above three cases, we get

$$f'(x) = 2|x| \text{ for every } x \text{ in } R$$

NOTE 1:-

The function f' may have a derivatives denoted by f'' which is defined at all points where f'' is differentiable . f'' is called the second derivatives of f

NOTE 2 :-

$\lim\limits_{h \to 0} \frac{f(x)-f(0)}{x-0}$ does not exist

If x< 0. Thus f does not have a derivative at zero ; even though

f is continuous at zero.

LEMMA 2

If f and g are both differentiable at x = a in R, then f+g , f-g and f .g are differentiable and have derivatives given by ,

I) $(f+g)'(a) = f'(a) + g'(a)$

II) $(f-g)'(a) = f'(a) - g'(a)$

III) $(fg)'(a) = f'(a)g(a) + f(a)g'(a)$

IV) If $g'(a) \neq 0$ then f/g is differentiable at a and has derivative given by

I) $\left(\frac{f}{g}\right)'(a) = \frac{g(a)+f'(a)-f(a)g'(a)}{[g(a)]^2}$

PROOF: (i) Let, we have

$(f+g)'(a) = f'(a) + g'(a)$

Taking LHS first ,

$(f+g)'(a) = f'(a) + g'(a)$

= RHS

II) $(f-g)'(a) = f'(a) - g'(a)$

Taking LHS first ,

$= f'(a) - g'(a)$

= RHS

III) Let h $= fg$ $\because x \neq a$

We get,

$h(x) - h(a) = f(x)g(x) - f(a).g(a)$

$$= f(x).g(x) - f(a).g(x) + f(a).g(x) - f(a).g(a)$$

And so,

$$\frac{h(x)-h(a)}{x-a} = \frac{f(x)-f(a)}{x-a}\, g(x) + f(a)\, \frac{g(x)-g(a)}{x-a}$$

Since , $\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

$$= f'(a) \lim_{x\to a} \frac{g(x)-g(a)}{x-a}$$

$$= g'(a)$$

By Lemma 1 , $\lim_{x\to a} g(x) = g(a)$

Hence by using the theorem on limits, h has a derivative at a and

$$h'^{(a)} = \lim_{x\to a} \frac{h(x)-h(a)}{x-a} = f'(a)g(a) + f(a)g'(a)$$

To prove (iv) , let $h = f/g$. Then we have,

$$\frac{h(x)-h(a)}{x-a}$$

$$= \frac{1}{g(x)g(a)} \left[g(a)\frac{f(x)-f(a)}{x-a} - f(a)\frac{g(x)-g(a)}{x-a} \right]$$

Since f (x) and g(x) are differentiable at a having the derivatives $f'(a)$ and $g'(a)$ and when x≠0,

$$\lim_{x\to a} g(x) = g(a)$$

We get from above

$$\lim_{x\to a} \frac{h(x)-h(a)}{x-a} = \frac{g(a)f'(a) - f(a)g'(a)}{g^2(a)}$$

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DISCUSSIONS

The derivative of any constant is zero. If $f(x) = x$, then $f'(x) = 1$ .by using (iii) proof repeatedly we see that x^n is differentiable and the derivative is nx^{n-1} for any integer n , when $x \neq 0$. Thus a polynomial is differentiable using in proof repeatedly. We see that every rational function is the differentiable except at the point where the denominator is zero.

For two functions f and g the composite function $h = gof$ is defined at each point a in the domain of f for which f (a) is in the domain of g and at such a point a, $h(a) = (gof)(a) = g[f(a)]$

LEMMA 3

If f is differentiable at a and g is differentiable at f(a) then $h = gof$ is differentiable at a and has the derivatives of

$$h'(a) = g'[f(a)]f'(a)$$

PROOF:-

Let f is differentiable at a so continuous & g be differentiable at b = f(a) , continuous also simultaneously

$$h = gof \text{ is continuous at } x = a$$

then, $\eta(x) = \frac{f(x)-f(a)}{x-a} - f'(a)$

Since $\lim_{x \to a} \frac{f(x)-f(a)}{x-a}$ exists then η(x) exists in a detected neighborhood of a & $\eta(x) \to 0$ as $x \to a$

Hence , if f is differentiable at a, we can write,

$$f(x) - f(a) = (x - a) [\ f'(a) + \eta(x)\ ]$$

Similarly, since g is differentiable at b = f(a)

We have ,

$$g(y) - g(b) = (y - b)[g'(b) + \gamma(y)]$$

Where

$\gamma(y) \to 0$, $y \to b$

Then, $\mathrm{h(x)} - \mathrm{h(a)} = (\,gof)x - (gof)a = g[f(x)] - g[f(a)]$

$= \; g(y) - g(b) = (y-b)[g^{'}(b) + \gamma(y)]$

$= \; [f(\mathrm{x}) - \mathrm{f(a)}]\ [\mathrm{g}^{'}(\mathrm{f\,(a)}\,) + \gamma\ (\mathrm{f(\,(x)\,)}\,]$

$= \; (x-a)\ \left[f^{'}(a) + \eta\ (x)\ [g^{'}(b) + \gamma(y)]\right]$

If $x \neq a$ we get,

$$\frac{\mathrm{h(x)} - \mathrm{h(a)}}{x-a} = [g^{'}(b) + \gamma(y)]\ [\ f^{'}(a) + \eta\ (x)\ ]$$

If lim x→ a then

$y = f(x) \to f(a) = b;$

Hence,

$\eta\ (x) \to 0$ & $\gamma\ (y) \to 0$ therefore,

$$h'(a) \;=\; \lim_{x \to a} \frac{\mathrm{h(x)} - \mathrm{h(a)}}{x-a}$$

$$=\; g^{'}(b)f^{'}(a)$$

NOTE :-

As inverse function gives the relationship between the derivatives of the inverse functions. If f is a one-one function on [a, b] intervals.

Then $q[f(x)] = x(a < x < b)$ where q is the inverse function for ‘f ’

EXAMPLES 2:–

If $f(\mathrm{x}) = \mathrm{x}$ as $\mathrm{x} \in \mathrm{R}$ & $\mathrm{f(x)} = \sin \mathrm{x}$ if x is $\mathrm{x} \in \mathrm{Q}$ prove that $f^{'}(0) = 1$

Ans :- We have

$f^{'}(0) =,\ \ \lim_{x \to a} \frac{\mathrm{f(x)} - \mathrm{f(0)}}{x-0}$

x is rational,

$f'(0) =, \quad \lim_{x\to a} \frac{x-0}{x-0} = 1$

x ∈ Q i.e. Irrational,

$f'(0) =, \quad \lim_{x\to 0} \frac{\sin x}{x} = 1$

$f'(0) =, \quad \lim_{x\to 0} \frac{\sin x}{x} = 1$

$f'(0) = 1$

EXAMPLES 3:

If $f(x) = e^{-|x|}$; ∀ x ∈ R then show that it is continuous at x = 0 but not differentiable at x = 0

Ans : f (0) = 0

When x > 0 , $f(x) = e^{-x}$

$\lim\limits_{x\to 0\pm} e^{\pm x} = \lim_{h\to 0} e^{\pm(0\pm h)}$

$= 1$

Then , $\lim_{x\to a} \frac{f(x)-f(0)}{x-0}$

$= \lim_{x\to 0} \frac{e^{-x}-1}{x}$

$= \lim\limits_{x\to 0} \left(1 - x + \frac{x}{1} + \frac{x^2}{2} \ldots\ldots..-1\right)/x = -1,$

When x < 0

$\lim_{x\to 0} \frac{f(x)-f(0)}{x-0}$

$= \lim_{x\to 0} \frac{e^{x}-1}{x} = \lim\limits_{x\to 0}(1 + x + \frac{x}{1} + \frac{x^2}{2} \ldots\ldots..-1/x)$

$= 1$

Hence $\lim_{x\to 0}\frac{\mathrm{f(x)-f(0)}}{x-0}$ does not exist at x = 0

REMARKS

As define the one – sided derivative of a function f. let f be defined n an interval [a , a+δ] for some δ> 0

DEFINITION

The right and left hand derivative of f at a is,

$$f'\pm = \lim_{x\to a\pm}\frac{\mathrm{f(x)-f(0)}}{x-0}$$

Also,

$$f'\pm = \lim_{h\to 0\pm}\frac{\mathrm{f(a+h)-f(a)}}{h}$$

REMARKS:

$f'(a)$ exists iff $f'+$(a) and $f'-$(a) exists and both are equal.

CONCLUSION

By the above lemma and examples we defined real valued functions defined on intervals. As f, g is continuous at a f(a) then, h = g of is continuous at a In this article we show that how much the methods real valued functions can be effective for solving on the problem of intervals. So we have found a series of solution for differential equations & we sure that these are more problems solves by above analysis.

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