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ORIGINAL ARTICLE

A STUDY OF AZIMUTHAL MODULATIONS OF THE SOLAR WIND INFLUENCE OF MAGNETIC FIELD

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ABSTRACT

In this paper, we study the evolution of azimuthal modulation of the solar wind with magnetic field. The evolution of azimuthal modulation of the solar wind is examined by numerical integration of the basic equations governing supersonic, super-Alfvenic , polytrophic one fluid flow in the equatorial plane of the Sun with radial as well as azimuthal magnetic fields . The numerical model is based on pseudo-conservative form of two dimensional time dependent system. The computations are started by introducing various sets of finite amplitude perturbations stationary with respect to the Sun. It is shown that for a properly chosen set of simultaneous perturbations of the radial velocity and temperature, quantitative agreement with the averaged observational results at 1AU can be obtained, including the advance of the density modulation with respect to those of velocity and temperature. It is further shown that modulations at 1AU depends on the relative azimuthal positions between the Sun and Earth.

KEYWORDS:

Azimuthal modulation, solar wind, azimuthal magnetic field, super-Alfvenic, finite amplitude perturbations.

INTRODUCTION

Many theoretical and experimental studies have been performed on the solar wind models as an energy source for the generation of very high pressure and temperature. An extensive literature of the solar wind models has been published [1] since the first detailed analysis by Parker [2]. We may classify these models through their spatial and time dependence as the one -

dimensional (referring to spherical symmetry) stationary, the one – dimensional time dependent, the two - dimensional (referring to models in which variables are functions of two spatial coordinates) stationary as well as the two - dimensional time dependent models. A number of physical processes which affect the solar wind have been examined, such as the effects of thermal conduction [3-4], rotation and magnetic fields viscosity non – thermal heating's as well as a combination of these effects [5-8]. Apart from such studies, in order to account for modulations observed in association with solar flares ,the time dependent one-dimensional solar wind has been examined, notably, in terms of similarity solutions [2, 9-12] and by a more vigorous numerical integration of the governing non linear equations [13-14],. However, in the one-dimensional models, the discussion of the effect of magnetic fields remains somewhat superficial.

The purpose of this paper is to describe a two-dimensional time dependent solar wind model with magnetic fields (as a function of the time t, radial r, and azimuthal Φ,coordinats), and to show that the physical characteristics of the observed average modulations [15] can be quantitatively reproduced by numerical integration of the basic equations of the problem. The details of formulations, numerical procedures and results are described together with discussions of physical significance and implication of the results.

FORMULATION OF PROBLEM

The 'average' (excluding shocks) modulations of the solar wind near 1AU [15] have the physical characteristic qualitatively similar to those for the density and velocity predicted theoretically by Matsuda and Sakurai [16] for a steady one fluid azimuthally dependent solar wind in the equatorial plane of the Sun with magnetic fields to the adiabatic, supersonic and super-Alfvenic flow approximations. We examine the evolution of azimuthal modulations with one fluid solar wind governed by the two- dimensional(r,Φ) hydro magnetic equations and an adiabatic energy equation. It has been shown that the major thermal effects such as conduction [4] and the non-thermal heating's [17] are confined within the radial distance, say 25R(R the solar radius). At 1AU observations show 90% of the energy is carried by the solar wind in the form of kinetic energy [1]. However, it has been shown [15] that agreement between one fluid adiabatic flow theory and observation is reasonably good. Thus we adapt the polytropic flow approximation, i.e., the adiabatic energy equation for the heliocentric radial distance of interest in this study. It was shown also by Weber and Daves [18] that the co- rotation of the solar wind with respect to the Sun ceases beyond the radius around 20R where the solar wind speed becomes equal to the local Alfven speed.

Hence in this study, we consider the modulations of the solar wind beyond the radial distance 30R in a system of stationary coordinates with the origin at the center of the Sun for convenience of interpretation of modulations at 1AU on the evolution and relative azimuthal positions between the Sun and the Earth, we consider modulations which are stationary in forms and with respect to the Sun at 30R for the time t≥1. It should be noted that such stationary features with respect to the Sun appear in the present system of coordinates as a function F(r¸Φ-Ωt)with = 2.87×10^{-6} rad s^{-1} denoting the angular velocity of solar rotation .It is assumed that before introduction of solar wind is characterized by a steady one dimensional solar wind similar to that given by Weber and Daves [18].

The flow governing equation in pseudo-conservation form is,

$$\frac{\partial U}{\partial t}+\frac{\partial F}{\partial r}+\frac{1}{r}\frac{\partial G}{\partial \phi}=S$$

Where U, F, G and S are all vectors.

The components of equation (1) can be identified with the equation of continuity, the radial and azimuthal equations of motion the radial and azimuthal equations of magnetic field induction, and the equation of energy conservation.

$$F \;= \begin{bmatrix} r^2\,\rho\, v_r \\ r^2\left[p+\,\rho\, v_r^2-\,\frac{1}{8\pi}\left(B_r^2-\,B_\emptyset^2\right)\right] \\ r^2\left(\rho v_r v_{\emptyset-}\,\frac{1}{4\pi}\,B_r\,B_\emptyset\right) \\ 0 \\ r(v_r B_\emptyset-\,v_\emptyset B_r) \\ r^2 v_r(\frac{\gamma}{\gamma-1}p+\,\frac{1}{2}\,\rho|v|^2)+\,\frac{r^2}{4\pi}\left(v_r B_\emptyset^2-\,v_\emptyset B_r B_\emptyset\right) \end{bmatrix}, \qquad (3)$$

$$G = \begin{bmatrix} r^2\, \rho v_\emptyset \\ r^2 \left(\rho v_r v_{\emptyset -} \frac{1}{4\pi} B_r B_\emptyset \right) \\ r^2 \left[p + \rho\, v_\emptyset^2 + \frac{1}{8\pi} (B_r^2 - B_\emptyset^2) \right] \\ - r(v_r B_\emptyset - v_\emptyset B_r) \\ 0 \\ r^2 v_\emptyset \left(\frac{\gamma}{\gamma - 1} p + \frac{1}{2} \rho |v|^2 \right) + \frac{r^2}{4\pi} (v_\emptyset B_r^2 - v_r B_r B_\emptyset) \end{bmatrix}, \tag{4}$$

$$S = \begin{bmatrix} 0 \\ \frac{1}{4\,\pi}\, r\, B_r^2 + 2rp - \rho\, G\, M + \rho\, r\, v_\emptyset^2 \\ \frac{1}{4\pi}\, r\, B_r\, B_{\,\emptyset} - \rho r\, v_r\, v_\emptyset \\ 0 \\ 0 \\ - v_r\, \rho\, G\, M_0 \end{bmatrix}, \tag{5}$$

Where ρ is the density, $v = (\, v_{r\,,} v_\emptyset)$ the velocity, $B = (B_{r\,,}\, B_\emptyset)$ the magnetic field induction, G the gravitational constant, $M\ is$ the solar mass. In deriving these equations we assume that the solar wind is a thermally perfect gas so that the internal energy is given by $\frac{p}{\rho\,(\gamma - 1)}$

where γ denoting the adiabatic index.

In order to examine the evolution of the magnetic field configuration, we obtain the following equation,

$$\frac{\partial\, A}{\partial\, t} = r\, (B_r\, v_\emptyset - B_\emptyset\, v_r) - C\,, \tag{6}$$

where A is the stream function and C is a determinate constant for each specific line of force. Equation (6) is obtained from the equation of magnetic field induction, after integration, with the use of the defining equation of the components of magnetic field in terms of A , i.e.

$$B_r = -\frac{1}{r^2}\frac{\partial A}{\partial \emptyset} , \qquad B_\emptyset = \frac{1}{r}\frac{\partial A}{\partial r} . \tag{7}$$

The numerical integrations are performed in a system of polar grid points ranging from $30\,R\ to\ 230\,R$ with radial grid distance of $R\ and\ \emptyset = 0^0$ to 180^0 with azimuthal grid distance of 2.5^0 . With this system of grid points the following time resolutions of the modulations at 1AU ($215R$) are obtained, i.e. for the radial velocity of $600\ km\ s^{-1}$ the radial time resolution of about $30\min(\approx \frac{R}{v_r})$ and in the azimuthal direction the time resolution of about $4.2h\ (\approx \frac{2.5^0}{\Omega})$ .The disparity of the radial and azimuthal grid points and resolutions are selected in consideration of the dominance of radial flow of the solar wind and the numerical stability of the results , after a number of trial computations.

NUMERICAL ANALYSIS

We use Rusanov scheme for solution of equation (1)-(5).The grid for the numerical calculations is chosen such that along the r and $\emptyset$ the spacing is equidistant. The indices i, j are used for numbering the grid points. This each grid point in space is defined by an ordered pair of in space is defined by an ordered pair of indices(i, j). For the time step along the t – coordinate the notation $t = n\,\tau$, is used $h_{1,}\ h_2$ are step sizes along $r\ and\ \emptyset$ then following set of explicit difference equations are derived.

$$\frac{U_{i,j}^{n+1} - U_{i,j}^{n}}{\tau} + \frac{F_{i+1,j}^{n} - F_{i-1,j}^{n}}{2h_1} + \frac{G_{i,j+1}^{n} - G_{i,j-1}^{n}}{2h_2}$$
$$= S_{i,j}^{n}$$
$$+ \frac{K_1}{2}\left[(\Omega_{11})_{i+\frac{1}{2},j}^{n} \frac{U_{i+1,j}^{n} - U_{i,j}^{n}}{h_1} - (\Omega_{11})_{i-\frac{1}{2},j}^{n} \frac{U_{i,j}^{n} - U_{i-1,j}^{n}}{h_1}\right.$$
$$\left. + (\Omega_{12})_{i+\frac{1}{2},j}^{n} \frac{U_{i+\frac{1}{2},j+\frac{1}{2}}^{n} - U_{i+\frac{1}{2},j-\frac{1}{2}}^{n}}{h_2} - (\Omega_{12})_{i-\frac{1}{2},j}^{n} \frac{U_{i-\frac{1}{2},j+\frac{1}{2}}^{n} - U_{i-\frac{1}{2},j-\frac{1}{2}}^{n}}{h_2}\right]$$
$$+ \frac{K_2}{2}\left[(\Omega_{22})_{i,j+\frac{1}{2}}^{n} \frac{U_{i,j+1}^{n} - U_{i,j}^{n}}{h_2} - (\Omega_{22})_{i,j-\frac{1}{2}}^{n} \frac{U_{i,j}^{n} - U_{i,j-1}^{n}}{h_2}\right.$$
$$\left. + (\Omega_{21})_{i,j+\frac{1}{2}}^{n} \frac{U_{i+\frac{1}{2},j+\frac{1}{2}}^{n} - U_{i-\frac{1}{2},j+\frac{1}{2}}^{n}}{h_1} + (\Omega_{21})_{i,j-\frac{1}{2}}^{n} \frac{U_{i+\frac{1}{2},j-\frac{1}{2}}^{n} - U_{i-\frac{1}{2},j-\frac{1}{2}}^{n}}{h_1}\right]$$

Where $\left(\Omega_{\alpha\beta}\right)_{i\pm\frac{1}{2},j}^{n} = \Omega_{\alpha\beta}\left(U_{i\pm\frac{1}{2},j}^{n}\right),\qquad (\alpha,\beta = 1,2)$,

$$U_{i\pm\frac{1}{2},j}^{n} = \frac{U_{i\pm\frac{1}{2},j}^{n} - U_{i,j}^{n}}{2}$$

The matrices $\Omega_{\alpha\beta}$ are defined as $\Omega_{\alpha\beta} = A_\alpha . A_\beta + C_{\alpha\beta},\ A_1 = \frac{d\,F}{d\,U}$, $A_2 = \frac{d\,G}{d\,U}$. $Where\ C_{\alpha\beta}$ are the matrices which include the influence of the artificial viscosity. The stability criterion of the difference scheme used is given by CFL as $\tau \le \frac{\alpha\,h}{\sigma\sqrt{2}}$, $\alpha \le 1$, σ is the spectral radius of the matrices A_1 and $A_2, h = h_1 = h_2$. For the present problem, the artificial viscosity matrices were chosen to be

$C_{12} = C_{21} = 0$, $C_{11} = C_{22} = \mu\, I$, I is the identity matrix. For the step sizes in $r\,, \emptyset -$ direction the following special choice was taken $h = h_1 = h_2$,$\mu = 2\mu_0 \frac{h^2}{\tau^2}$, $\mu_0 =$ Constant

The numerical results are discussed in the next section.

NUMERICAL RESULTS

The solutions of the algebraic equations which result from the integrals of the governing equations examine the problem [18]. More precisely the algebraic equations are solved for a set of conditions given at 1AU , such as v_{rE} ,ρ_E ,T_E (so that p_E), B_{rE} , and $v_{\emptyset E}$, where the subscript E refers to the values at 1AU . Such determinations for a given set of 1AU values lead to a set of values γ and r_c ,

Where r_c is the radius at which the radial velocity becomes equal to the local Alfven speed , we then selected the solutions which give the conditions similar to the undisturbed state of solar wind of and the value of γ close to $\frac{5}{3}$ with r_c, less than $30R$,i.e.

$$v_{rE} = 400\frac{km}{s} \quad , \quad \rho_E = 1.4 \times 10^{-23} g\, cm^{-3}$$

$$T_E = 5 \times 10^4\, K \quad , \quad B_{rE} = 3 \times 10^{-5} G ,$$

$$v_{\emptyset E} = 2\, km\, s^{-1} \quad , \quad r_c = 19.4\ R,$$

$$\gamma = 1.6227 \quad , \qquad (8)$$

Where ρ_E corresponds to the number density of 7 particles cm^{-3} with the mean molecular weight 2×10^{-24} g.

Numerical computations are performed for various sets of finite amplitude perturbations, such as modulations of ρ , T and v_r in different combinations with different azimuthal extents and amplitudes as well as phases. After a number of computations it is found that the average solar wind modulations can be reproduced closely by the following set of perturbations,

$$v_r = v_{r_0}\left[\frac{5}{4} + \frac{1}{4}\sin(\emptyset, t)\right], \qquad \Omega\, t < \emptyset < \Omega\, t + \frac{\pi}{2},$$

$$T = T_0\left[\frac{3}{2} + \frac{1}{2}\sin(\emptyset, t)\right], \qquad \Omega\, t < \emptyset < \Omega\, t + \frac{\pi}{2}, \qquad (9)$$

Where

$$\sin(\emptyset , t) = \sin\left[6 (\emptyset - \Omega t) - \frac{\pi}{2}\right] , \qquad (10)$$

$v_{r_0} = 346\ km\ s^{-1}$ and $T_0 = 6.49 \times 10^5\ K$ are the values at $r = 30\ R$ of the initial steady state. Physically, equations (9) and (10) represent simultaneous sinusoidal modulations of v_r and T of the azimuthal extent 60^0, having the amplitudes 1.5 $v_{r_0}$and $2\ T_0$ at $30\ R$. The cause of these simultaneous enhancements of v_{r_0} and T_0 may be identified with a large photospheric region of activity, since such a region is characterized

by open magnetic field lines with higher coronal temperature which subsequently can lead to higher temperature and flow velocity at $30\ R$.

The successive evolution of the perturbations given by equations (9) and (10) has shown in figures 1, 2 and 3, in terms of the modulations of v_r and the magnetic field line configurations. The perturbations are introduced at $t \geq 0$ centered at $\emptyset = 30^0$, and in figure 1 the figures for $t = 0$ refer to the initial state. By assumption the v_r perturbation maintains the same form at $30\ R$ and rotates in the figures with elapsing time due to the present choice of coordinates. It can be seen readily in figure 1that the initial developments of the modulations are mainly in the radial direction, followed by subsequent gradual but complex azimuthal modulations as seen in figures 2 and 3 for $t > 50\ h$. It can be noticed in figures 2 and 3 that the modulations reach $1\ AU$ about $t = 75h$ at$\emptyset = 45^0$, then for $t > 125\ h$ the modulations evolve to a quasi-stationary state, and for $\emptyset > 75^0$ the modulations are purely azimuthal and stationary.

The dependence of modulation at $1\ AU$ on the relative positions between the Sun and the Earth are shown in figure 4 in which evolutions of the modulations of radial velocity; v_r , number density N and temperature T seen at different azimuthal angles $\emptyset$ are shown. At $\emptyset = 45^0$ the modulations at $1\ AU$ resemble those of one dimensional time dependent solutions discussed by Hundhausen while at $\emptyset = 75^0$ the physical characteristics of modulations resemble those of the two-dimensional stationary solutions given by Matsuda and Sakurai [16]. The most physically significant result shown in figure 4 is that the density modulations start in advance of those of the radial velocity and temperature for all $\emptyset$, i.e. approximately $5\ h$ at $\emptyset = 45^0, 10\ h$ at $\emptyset = 60^0$

and 20 h at $\emptyset = 75^0$. Such as advance of the density signal has not been obtained previously for modulations introduced near the Sun either in the one-dimensional time dependent solutions or in the two – dimensional stationary solutions by Matsuda and Sakurai [16]. The results shown in figure 4 also give the time difference between the start and the peak of density modulation as $10\ h$ at $\emptyset = 45^0$, $15\ h$ at $\emptyset = 60^0$, and $34\ h$ at $\emptyset = 75^0$. Further figure 4 shows that the relative time differences between the peaks of density and radial velocity modulations are $10\ h$ at $\emptyset = 45^0$, $20\ h$ at $\emptyset = 60^0$, and $25h$ at $\emptyset = 75^0$, and the similar time difference between the density and temperature being $7h$ at $\emptyset = 45^0$, $8h$ at $\emptyset = 60^0$ and $11h$ at $\emptyset = 75^0$.

The average modulations are reproduced by the results shown in figure 4 at $60^0 < \emptyset < 75^0$, in quantitative agreement with respect to the amplitudes of modulations as well as the time differences between the peaks of modulations of density , radial velocity and temperature , including the advance of the density signal with respect to the velocity and temperature of $10 - 20\ h$. In addition the peaks of temperature modulations are attained $10 - 15\ h$ ahead of the velocity peaks in comparison with approximate $6h$ in the average modulations.

In other words, the present results imply that the observed average modulations represent at mixture of azimuthal and evolutional modulations, since purely azimuthal (steady) modulations at $1\ AU$ for $\emptyset > 75^0$ show less satisfactory quantitative agreement with the average modulations , particularly with the smaller amplitude and a larger advance of the density signal.

CONCLUSION

It is shown in this paper that the observed ‘average’ modulations at $1\ AU$ can be reproduced in quantitative agreement by a set of simultaneous perturbations of the radial velocity and temperature at $30\ R$. It is also shown that the ‘average’ modulations at $1\ AU$ can be a mixture of azimuthal and evolutional modulations and that the modulations at $1\ AU$ depend on the relative positions between the Sun and the Earth. Gosling, noted that the advance of the density signal with respect to radial velocity and temperature has not been obtained in (previous) theoretical models, and these authors attributed the possible cause to the presence of a density enhancement ahead of the possible velocity modulation near the Sun in the basic slow stream[15].

It was also noted that the modulations observed at $1\ AU$ are not exactly reproduced at every solar rotation [15]. This result is again in agreement with the present results that the average modulations contain the evolutional effect and also with the fact that the mean life time of coronal structures, prominences, and large scale solar magnetic fields in low heliographic latitudes are of the order of one rotation [19]. In other words, we make consider that the modulations observed at $1\ AU$ always contain some evolutional consequences. The results of present study then clearly illustrate the importance or the examinations of time dependent solutions. The present results also illustrate the limitations of the linear perturbation approach such as the scheme proposed by some pioneer workers [20] for the identification of the casual relationships between the modulations observed at $1\ AU$ and those near the Sun through super positions of linear perturbations.

In summery, we must stress the importance of nonlinear time dependent study and the basic differences between the one – dimensional and two – dimensional solar wind models. It was pointed out by [21] that large azimuthal modulations can result in the two – dimensional model because of the difference in the radial dependences of the variables B_r and $v_\emptyset$ between the one – dimensional and two – dimensional steady state solutions. It is shown in this study that a full account of nonlinear time evolution can lead to still another effect such as the advance of density signal ahead of other modulations. However we must note the limited scope of the results of the present numerical study, apart from the full account of two – dimensional hydromagnetics. Under realistic circumstances, the modulations observed at $1\ AU$ must be subject to complex physical processes, i.e., the perturbations in the solar corona are subject to thermal conduction near the Sun [4] then to various possible non-thermal heating's [17,22] even near the Earth [7] including small scale instabilities such as perturbations in magnetic fields [23,24].

Nevertheless, we may state that the resent study reflects some of the physical reality of the actual modulations of the solar wind, and illustrate the difficulties of a unique identification between the observed modulations at $1\ AU$ and specific physical causes near the Sun, needless to say with particular coronal structures.

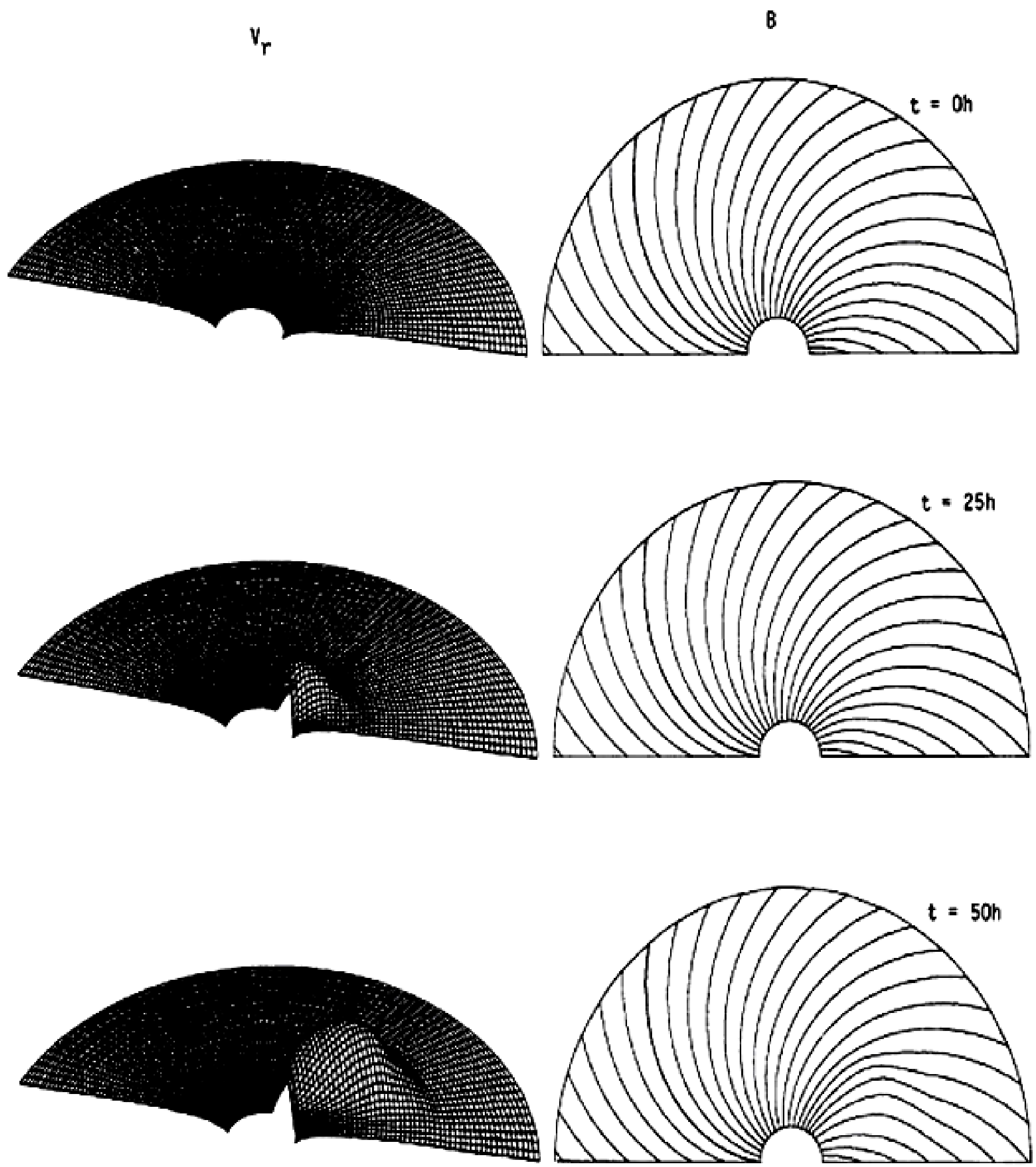


Fig. 1. The evolutions of the radial velocity v_r and the magnetic field configuration denoted by B in response to stationary azimuthal modulations with respect to the Sun introduced at 30 R at $t \geqslant 0$. The domain covers from 30 R to 230 R, and t refers to the lapsed time.

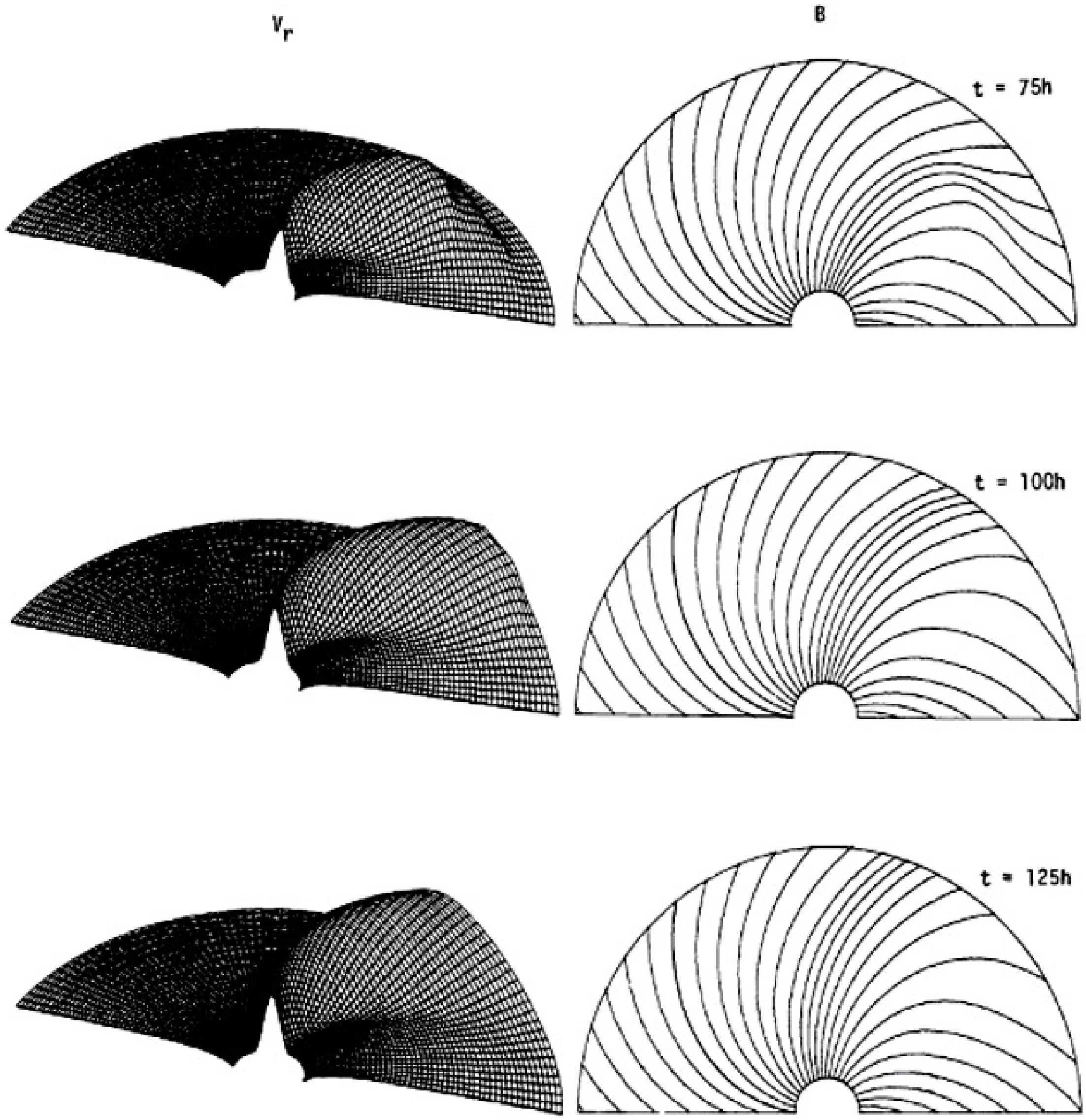


Fig. 2. The evolutions of the radial velocity v_r and the magnetic field configuration denoted by B in response to stationary azimuthal modulations with respect to the Sun introduced at 30 R at $t \geqslant 0$; t refers to the lapsed time.

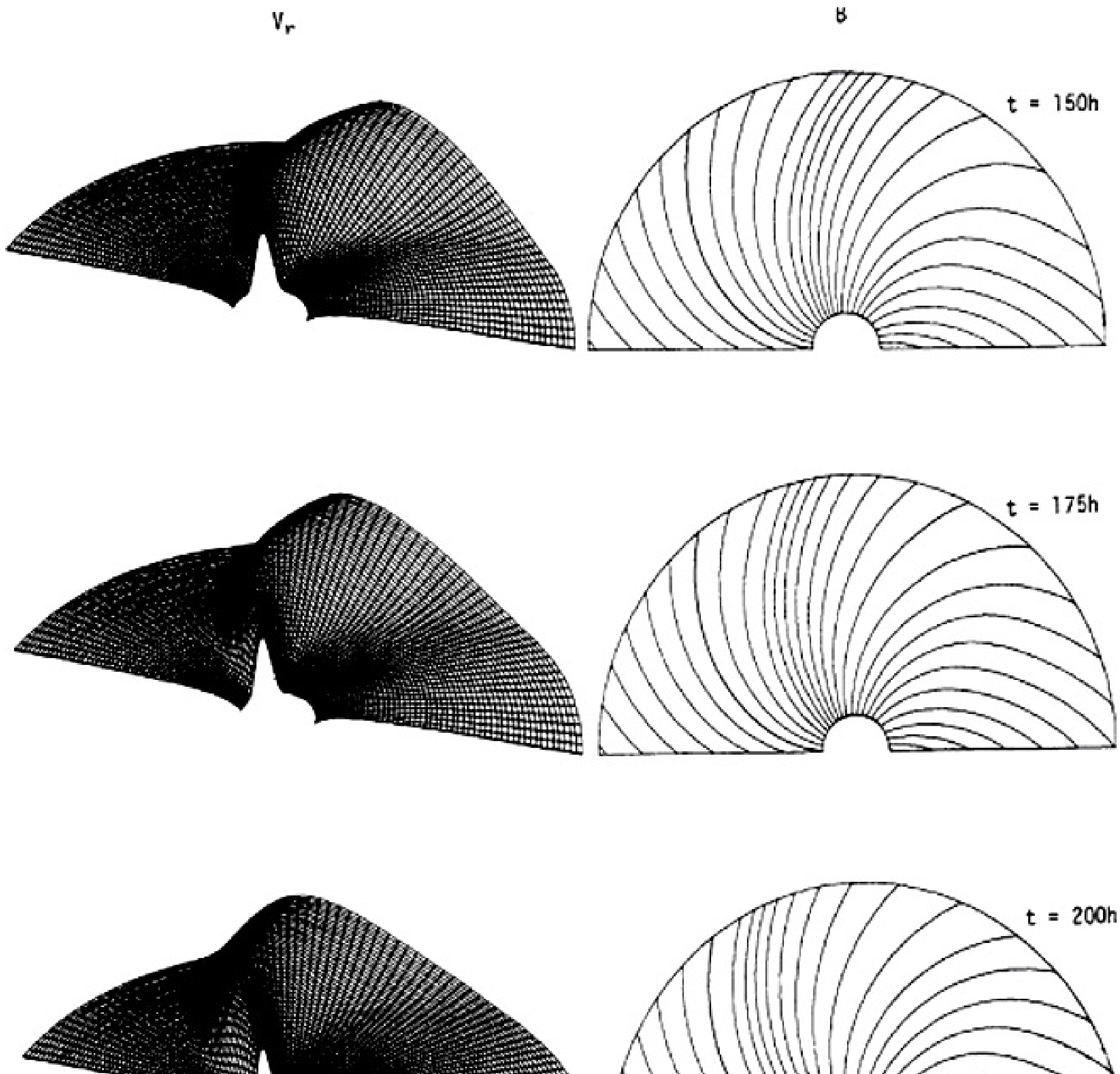
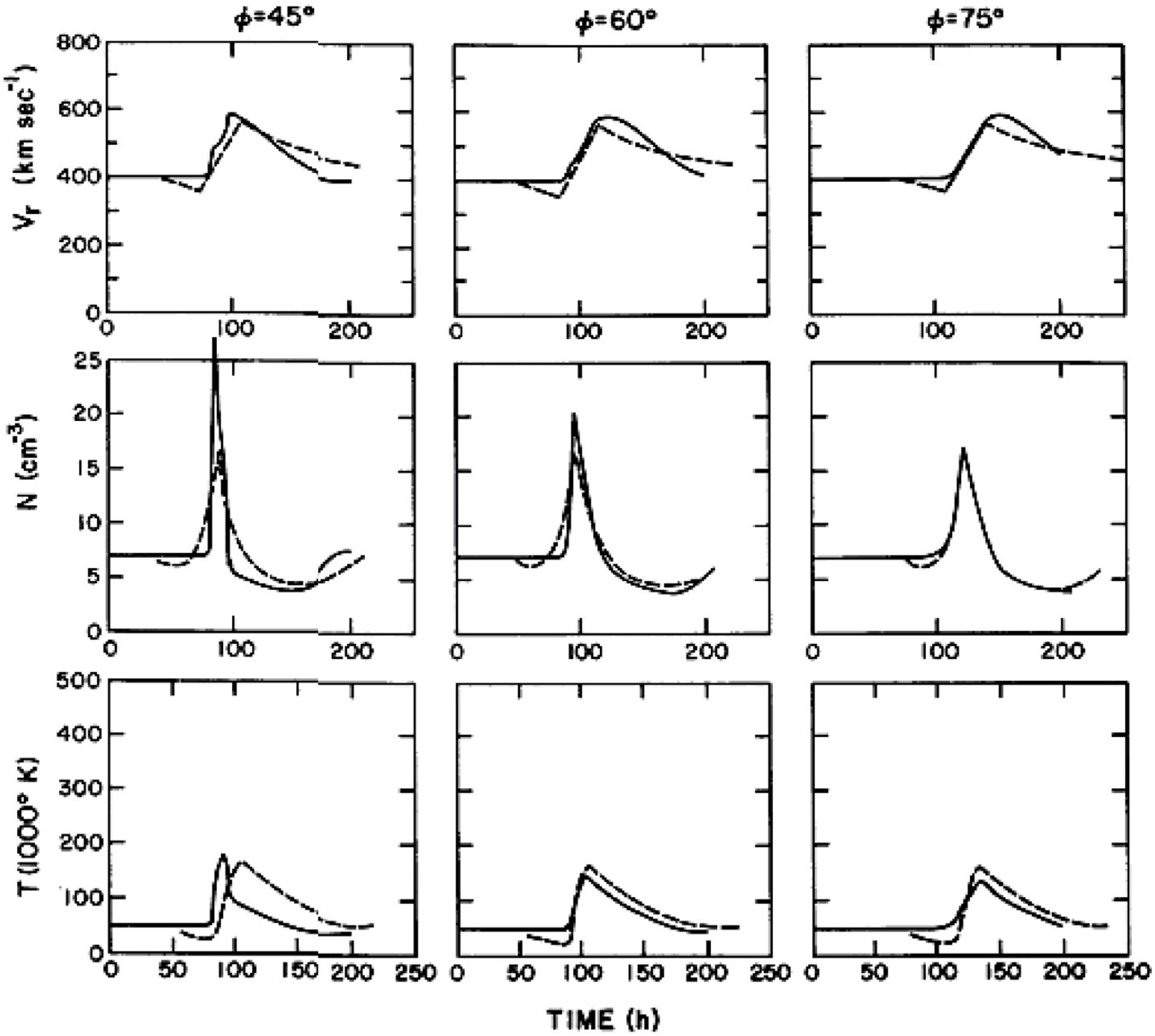


Fig. 3. The evolutions of the radial velocity v_r and the magnetic field configuration denoted by B in response to stationary azimuthal modulations with respect to the Sun introduced at 30 R at $t \geq 0$; t refers to the lapsed time.



]Fig. 4. The variations of tle responses at 1 AU for different azimuthal angles ϕ; v_r is the radial velocity, N the number densiy, T the temperature and t refers to the lapsed time. The dashed curves represent the average modulations by Gosling *et al.* (1972) for reference.

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