Golden Research Thoughts
ISSN 2231-5063
Volume-3, Issue-5, Nov-2013 Available online at www.aygrt.isrj.net

TRAJECTORY CONTROL OF TWO LINK ROBOTIC MANIPULATOR USING PID

Manjeet And Pooja Khatri

HCTM Technical campus Kaithal, India
HCTM Technical campus Kaithal, India

Abstract: This paper presents the design and implementation of PID controller for the general form of two-link planar robotic manipulator. Also for comparative analysis, proportional-derivative (PD), is implemented on the same system under same conditions. The comparative analysis of the results demonstrates that the PID controller is best among all the conventional controllers and the simulation results confirm that the system can track the desired trajectory. Computer simulation results on a two-link planar robotic manipulator are presented to show tracking capability and effectiveness of the proposed control scheme. The simulations have been carried out using Matlab.

Key words: - Two-link robotic manipulator, PID Controller, PD Controller,Matlab simulink tool.

INTRODUCTION

It is well established that robotic manipulators are highly dynamically coupled, time-varying, and highly nonlinear systems that are extensively used in industrial applications. The robotic manipulators are generally subjected to both structured and unstructured uncertainties [1, 2], which makes the accurate position control of the robotic arms a challenging task. The end effectors of the robotic manipulators are to follow some desired trajectories as close as possible. Therefore, trajectory tracking problem is the most significant and fundamental task in control of robotic manipulators. With the use of the robots in critical applications like medical and other sensitive areas, the precise control of the robot arms has become an essential requirement.

Motivated by such control requirements, for practical and complex control problem of robotic manipulators, in the past decades, many research contributions have been reported on robotic control schemes such as such as proportional-integration-derivative (PID) control [3],PD control, PI control[3], feed-forward compensation control [4], adaptive control [6], variable structure control [7], computed torque control[2,12]. The conventional control techniques are inadequate under large uncertainty and/or unpredictable variations in system parameters and structures. Most conventional control techniques require a precise mathematical model, which is not always possible but tuning of the controller highly required for getting the desired result.

In recent years, much attention has been paid to the use of PID control for robotic manipulators. The survey on PID control for robotic manipulators can be found in references cited therein.

In this paper, a PID control scheme is developed and implemented for trajectory tracking problem of two-link robotic manipulator. The performance of PID control is compared with that of conventional PD Controls [2].

MODEL OF ROBOTIC MANIPULATOR

According to Lagrange theory [2, 18], dynamical equations of robotic manipulator can be described by the non linear differential equation [1, 2, 18].

$$M(q)\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{bmatrix} + \begin{bmatrix}-m_2a_1a_2(2\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_2^{\,2})\sin\theta_2\\ m_2a_1a_2\dot{\theta}_1^{\,2}\sin\theta_2\end{bmatrix} + \begin{bmatrix}(m_1 + m_2)ga_1\cos\theta_1 + m_2ga_2\cos(\theta_1 + \theta_2)\\ m_2ga_2\cos(\theta_1 + \theta_2)\end{bmatrix} = \begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} \quad (1)$$

Where

$$M(q) = \begin{bmatrix} (m_1 + m_2)a_1{}^2 + m_2 a_2{}^2 + 2m_2 a_1 a_2 \cos\theta_2 & m_2 a_2{}^2 + m_2 a_1 a_2 \cos\theta_2 \\ m_2 a_2{}^2 + m_2 a_1 a_2 \cos\theta_2 & m_2 a_2{}^2 \end{bmatrix}$$

The manipulator dynamics are in the standard form [2, 18]

$$M(q)\ddot{q} + V(q,\dot{q}) + G(q) = \tau \qquad (2)$$

where $M(q)$is the n × n symmetric positive definite manipulator inertia matrix, $V(q,\dot{q})$is the n ×1vector of centrifugal and Coriolis torques, $G(q)$is the n ×1 vector of gravitational torque, τ is the n ×1 vector of joint torque, q is the n ×1 vector of the joint displacement (angular position), and $\ddot{q}$ and $\dot{q}$are the n ×1vectors of the joint acceleration and velocity terms, respectively [2].

The units of elements of $M(q)$corresponding to revolute joint variables $q_i = \theta_i$are kg-m^2. The units of the elements of $M(q)$corresponding to prismatic joint variables $q_i = d_i$are kilograms. The units of elements of $V(q,\dot{q})$and $G(q)$corresponding to revolute joint variables are kg- m^2/s^2. The units of elements of $V(q,\dot{q})$ and $G(q)$corresponding to prismatic joint variables are kg-m/s^2 [2].

Now the state-space formulations of the arm dynamics may be obtained by defining the position/velocity state $x \in R^{2n}$as [2, p145]

$$x = [q^T \ \dot{q}^T]^T$$

Equation (2) may be written as

$$\frac{d}{dt}\dot{q} = -M^{-1}(q)[V(q,\dot{q}) + G(q)] + M^{-1}(q)\tau \qquad (3)$$

Now, we may directly write the position/velocity state-space representation

$$\dot{x} = \begin{bmatrix} \dot{q} \\ -M^{-1}(q)N(q,\dot{q}) \end{bmatrix} + \begin{bmatrix} 0 \\ -M^{-1}(q) \end{bmatrix}\tau \qquad (4)$$

Which is in the form of $\dot{x} = f(x,u,t)$ with $u(t) = \tau(t)$.

An alternative linear state equation of the form $\dot{x} = Ax + Bu$ may be written as

$$\dot{x} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ I \end{bmatrix} u \qquad (5)$$

With control input defined by

$$u(t) = -M^{-1}(q)N(q,\dot{q}) + M^{-1}(q)\tau \qquad (6)$$

the control law

$$\tau = N(q) - M(q)u + M(q)\ddot{Q}_d \qquad (7)$$

where u = Control signal

$\ddot{Q}_d$ = Desired trajectory

Now In reality, a robot arm is always affected by friction and disturbances. Therefore, we shall generalize the arm model we have just derived by writing the manipulator dynamics as

$$M(q)\ddot{q} + V(q,\dot{q}) + F(\dot{q}) + G(q) + \tau_d = \tau \quad (8)$$

with q the joint variable *n*-vector and τ the *n*-vector of generalized forces. $M(q)$ is the inertia matrix, $V(q,\dot{q})$ the Coriolis/centripetal vector, and $G(q)$ the gravity vector. We have added a friction term

$$F(\dot{q}) = F_v\dot{q} + F_d \qquad (9)$$

With F_v the coefficient matrix of viscous friction and F_d a dynamic friction term. Also added is a disturbance τ_d, which could represent, for instance, any inaccurately modeled dynamics.

We shall sometimes write the arm dynamics as

$$M(q)\ddot{q} + N(q,\dot{q}) + \tau_d = \tau \qquad (10)$$

Where

$$N(q,\dot{q}) \equiv V(q,\dot{q}) + F(\dot{q}) + G(q) \qquad (11)$$

Represent non linear terms.

PROPERTIES OF ROBOTIC MANIPULATOR[2]

Let us examine the structure and properties of each of the terms in the robot dynamics equation.

Property 1:

The inertia matrix $M(q)$ is a positive definite symmetric matrix bounded by $\mu_1 I \le M(q) \le \mu_2 I$, where μ_1, μ_2 , are known positive constants[1,2,29].

$$\mu_1 I \le M(q) \le \mu_2 I \quad (12)$$
$$m_1 \le M(q) \le m_2 \quad (13)$$

Property 2:

The matrix $\dot{M}(q) - 2V_m(q,\dot{q})$ is skew-symmetric. This implies

$$\dot{M}(q) = V(q,\dot{q}) + V(q,\dot{q})^T \quad (14)$$

$V(q,\dot{q})$ is quadratic in $\dot{q}$

$\|V(q,\dot{q})\| \le v_b \|\dot{q}\|^2$ (15) $V(q,\dot{q}) = V_m(q,\dot{q})q$

(16)

Where

$v_b(q)$ is a known scalar function, and for a revolute arm, v_b is a constant independent of q. $\|\cdot\|$ is any appropriate norm.

Property 3:

The viscous friction F_v may be assumed to have the form $F_v = diag\{v_1\}$, with v_1 being known constant coefficients. The dynamic friction F_d may be assumed to have the form $F_d(\dot{q}) = K_d\, sgn(\dot{q})$, with $K_d = diag\{k_i\}$ being known constant coefficients. Thus, the bound on the friction terms may be assumed to be of the form

$$F(\dot{q}) = F_v\dot{q} + F_d(\dot{q}) \quad (17)$$
$$F_v = diag\{v_1\} \quad (18)$$
$$F_d(\dot{q}) = K_d\, sgn(\dot{q}), \text{ with } K_d = diag\{k_i\} \quad (19)$$
$$\|F_v\dot{q} + F_d(\dot{q})\| \le v\|\dot{q}\| + k \quad (20)$$

Property 4:

A bound on the gravity term may be derived for any given robotic manipulator

$$\|G(q)\| \le g_b \quad (21)$$

Where $\|\cdot\|$ is any appropriate vector norm and g_b is a scalar function that may be determined for any given robotic manipulator.

Property 5:

The term τ_d which could represent inaccurately modeled dynamics, and so on. We shall assume that it is bounded so that

$$\|\tau_d\| \le d, \quad (22)$$

Where d is a scalar constant that may be computed for a given arm and $\|\cdot\|$ is any suitable norm.

Property 6:

The robot dynamical equation enjoys one last property that is linear in the parameters. This is important, since some or all of the parameters may be unknown; thus the dynamics are linear in the unknown terms [2,29]. This property may be expressed as

$$M(q)\ddot{q} + V(q,\dot{q}) + F_v(\dot{q}) + F_d(\dot{q}) + G(q) \quad (23)$$
$$= M(q)\ddot{q} + N(q,\dot{q}) \equiv W(q,\dot{q},\ddot{q})\varphi \quad (24)$$

with φ the parameter vector and $W(q,\dot{q},\ddot{q})$ a matrix of robot functions depending on the joint variables, joint velocities, and joint accelerations. This matrix may be computed for any given robot arm and so is known. Note that the disturbance τ_d is not included in this equation.

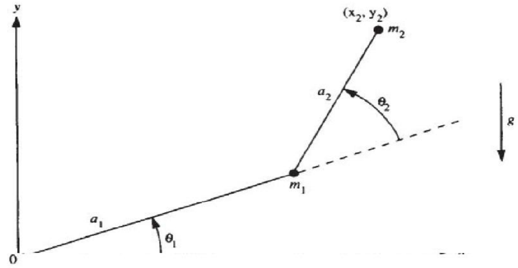

Fig 1 Two link Robotic Arm [2]

CONTROLLER DESIGN AND RESULTS

The following are the common parameters used in the simulation of all control laws,

TABLE I. SYSTEM PARAMETER

	Mass(m) In kg	***Length(l) In meter***	***Position or initial angle(q)***	$\dot{q}$	***Desired trajectory***
Link 1	$m_1 = 1$	$l_1 = 1$	$\theta_1 = 0^\circ$	$\dot{\theta}_1 = 0^\circ$	$\theta_{d1} = \sin(\pi t)$
Link 2	$m_2 = 1$	$l_2 = 1$	$\theta_2 = 0^\circ$	$\dot{\theta}_2 = 0^\circ$	$\theta_{d2} = \cos(\pi t)$

Now the error is given as

$$e = \begin{bmatrix} e_{11} \\ e_{21} \end{bmatrix}, \ \dot{e} = \begin{bmatrix} \dot{e}_{11} \\ \dot{e}_{21} \end{bmatrix}. \quad (25)$$

where

$$e_{11} = \theta_{d1} - \theta_1, \quad (26)$$
$$e_{21} = \theta_{d2} - \theta_2, \quad (27)$$
$$\dot{e}_{11} = \dot{\theta}_{d1} - \dot{\theta}_1, \quad (28)$$
$$\dot{e}_{21} = \dot{\theta}_{d2} - \dot{\theta}_2. \quad (29)$$

The torque is given by the equation as [2]

$$\tau = N(q) - M(q)u + M(q)\ddot{\theta}_d \quad (30)$$

$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$ = torque

τ_1= Torque for link 1.

τ_2= Torque for link 2.

$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ = Control signal.

u_1= Control signal for link 1.

u_2= Control signal for link 2.

And acceleration ($\ddot{\theta}$) can be calculated by the equation given as [2,18]

$$\ddot{\theta} = -M^{-1}(q)[N(q)] + M^{-1}(q)\tau \quad (31)$$

Now simulation for each control laws is given below one by one as

Design of PD Controller

The formula used for finding the control signal 'u' is given as

$$u = K_p e(t) + K_d \dot{e}(t) \quad (32)$$

Where K_p = 12 and K_d = 18

Now the results for PD control of robotic 2 link arm are shown in Figs. 2 to 5.

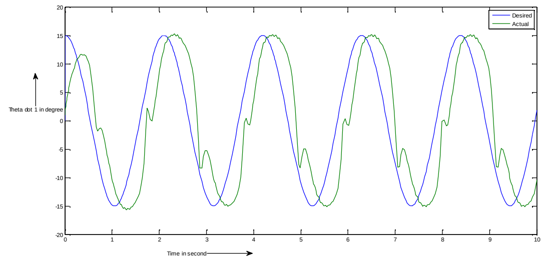

Fig.2 Velocity tracking for joint-1($\dot{\theta}_1$)

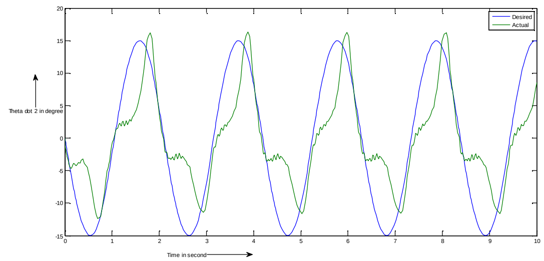

Fig.3 Velocity tracking of joint-2($\dot{\theta}_2$)

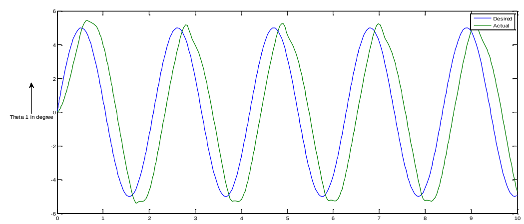

Fig.4 Position tracking of joint-1(θ_1)

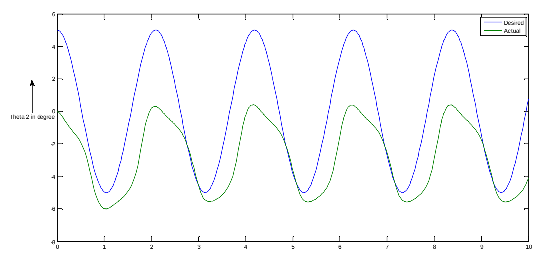

Fig.5 Position tracking of joint-2(θ_2)

Design of PID Controller

The formula used for finding the control signal (u) is given as

$$u = K_p e(t) + K_i \int_0^t e(t)dt + K_d \dot{e}(t) \qquad (33)$$

where $K_p = 50$, $K_i = 2$ $K_d = 60$

Now the results for PID control of robotic 2 link arm are shown in Figs. 6 to 9.

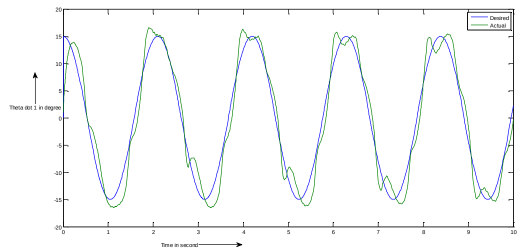

Fig.6 Velocity tracking for joint-1($\dot{\theta}_1$)

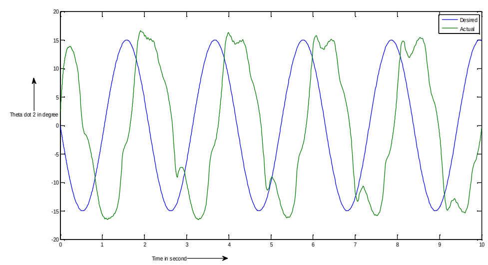

Fig.7 Velocity tracking of joint-2($\dot{\theta}_2$)

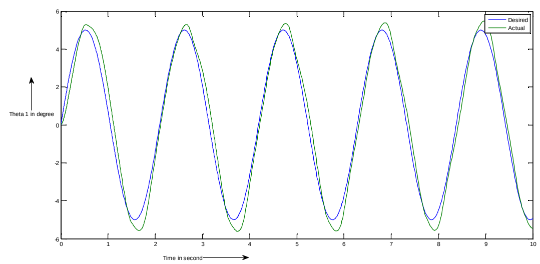

Fig.8 Position tracking of joint-1(θ_1)

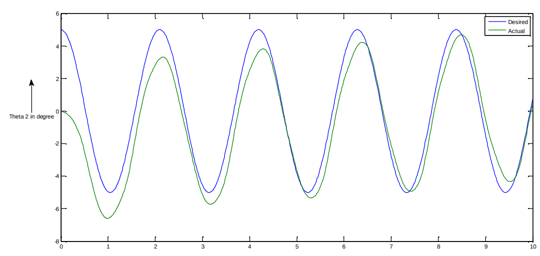

Fig.9 Position tracking of joint-2(θ_2)

CONCLUSION

The performance of two-link robotic manipulator is investigated with PD, PID control. The PID controller resulted in the best performance and very effective and accurate trajectory tracking capability as compared to PD controller. Also the response with PID controller was having reduced oscillations about the desired trajectory as compared to PD controller.

REFERENCES

I. Limin Peng and Peng-Yung Woo, “Neural-Fuzzy Control System for Robotic Manipulator” IEEE Control System Magazine, Feb 2002.
II. F.L. Lewis, C.T. Abdallah, and D.M. Dawson, Control of Robot Manipulators, New York: Macmillan, 1993.
III. J.Y.S. Luh, Conventional controller design for Industrial robots-a tutorial, IEEE Trans. Systems Man Cybernet. 13 (1983) 298–316.]
IV. P.K. Khosla, T. Kanade, Experimental evaluation of nonlinear feedback and feedforward control schemes for manipulator, Int. J. Robot. Res. 7 (1988) 18–28.

V. N. Selvaganesan, Prabhu Jude Rajendran, S.Renganathan,“Fuzzy Based Robust Controller Design for Robotic Two-Link Manipulator” International Conference, August 3-5 2004, Kota Kinabalu, Sabah, Malaysia

VI. J.J.E. Slotine, W. Li, Adaptive manipulator control: a case study, IEEE Trans. Automat. Control 33 (1988) 995–1003.

VII. Mark W.Spong and M.Vidyasagar, ”Robot Dynamics and control” John Wiley & Sons,2004.

VIII. W.T. Miller, F.H. Glanz, L.G. Kraft, Application of a general learning algorithm to the control of robotic manipulators ,Int. J. Robot. Res. 6 (1987) 84–98.

IX. T. Ozaki, T. Suzuki, T. Furuhashi, S. Okuma,Y. Uchikawa, Trajectory control of robotic manipulator using neural networks, IEEE Trans. Ind. Electron. 38 (1991) 195–202.

X. B.S. Chen, H.J. Uang, C.S. Tseng, Robust tracking enhancement of robot systems including motor dynamics: a fuzzy-based dynamic game approach, IEEE Trans. Fuzzy Systems 6 (1998) 538–552.

XI. S.Y. Yi, M.J. Chung, A robust fuzzy logic controller for robot manipulators with uncertainties, IEEE Trans. Systems Man Cyber net. 27 (1997) 706–713

XII. Zuoshi Song, Jianqiang Yi, Dongbin Zhao, Xinchun Li, “A computed torque controller for uncertain robotic manipulator systems: Fuzzy approach” Fuzzy Sets and Systems 154 (2005) 208–226,China.

XIII. L. Zadeh, Fuzzy sets, Inform. Control 8 (1965) 338–353.

XIV. L.X. Wang, Stable adaptive fuzzy control of nonlinear systems, IEEE Trans. Fuzzy Systems 1 (1993) 146–155.

XV. S. Commuri and F.L. Lewis, “Adaptive-fuzzy logic control of robot manipulators” in Proc. 1996 IEEE Int. Conf. Robotic Automation, Minneapolis, MN, Apr.1996.

XVI. T. Corbet, N. Sepehri et al., “Fuzzy control of a class of hydraulically actuated industrial robots,” IEEE Trans. Contr. Syst. Technol., vol. 4, no. 4, 1996

XVII. H.A. Malki, D. Misir et al., “Fuzzy PID control of a flexible-joint robot arm with uncertainties from time-varying loads,” IEEE Trans. Contr. Syst. Technol., vol. 5, no. 3, 1997

XVIII. S. Purwar, I.N. Kar, A.N. Jha, Adaptive control of robot manipulators using fuzzy logic systems under actuator constraints, Fuzzy Sets and Systems; in press. http://www.elsevier.com/locate/fss.

XIX. J. Ross, “Fuzzy logic with engineering applications, ”John wiley & Sons., India.