TRAJECTORY CONTROL OF TWO LINK ROBOTIC MANIPULATOR USING PID

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Abstract: This paper presents the design and implementation of PID controller for the general form of two-link planar robotic manipulator. Also for comparative analysis, proportional-derivative (PD), is implemented on the same system under same conditions. The comparative analysis of the results demonstrates that the PID controller is best among all the conventional controllers and the simulation results confirm that the system can track the desired trajectory. Computer simulation results on a two-link planar robotic manipulator are presented to show tracking capability and effectiveness of the proposed control scheme. The simulations have been carried out using Matlab.

Key words: - Two-link robotic manipulator, PID Controller, PD Controller, Matlab simulink tool.

INTRODUCTION

It is well established that robotic manipulators are highly dynamically coupled, time-varying, and highly nonlinear systems that are extensively used in industrial applications. The robotic manipulators are generally subjected to both structured and unstructured uncertainties [1, 2], which makes the accurate position control of the robotic arms a challenging task. The end effectors of the robotic manipulators are to follow some desired trajectories as close as possible. Therefore, trajectory tracking problem is the most significant and fundamental task in control of robotic manipulators. With the use of the robots in critical applications like medical and other sensitive areas, the precise control of the robot arms has become an essential requirement.

Motivated by such control requirements, for practical and complex control problem of robotic manipulators, in the past decades, many research contributions have been reported on robotic control schemes such as such as proportional-integration-derivative (PID) control [3], PD control, PI control[3], feed-forward compensation control [4], adaptive control [6], variable structure control [7], computed torque control[2,12]. The conventional control techniques are inadequate under large uncertainty and/or unpredictable variations in system parameters and structures. Most conventional control techniques require a precise mathematical model, which is not always possible but tuning of the controller highly required for getting the desired result.

In recent years, much attention has been paid to the use of PID control for robotic manipulators. The survey on PID control for robotic manipulators can be found in references cited therein.

In this paper, a PID control scheme is developed and implemented for trajectory tracking problem of two-link robotic manipulator. The performance of PID control is compared with that of conventional PD Controls [2].

MODEL OF ROBOTIC MANIPULATOR

According to Lagrange theory [2, 18], dynamical equations of robotic manipulator can be described by the non linear differential equation [1, 2, 18].

\[
M(q) \dot{\theta} + \begin{bmatrix}
-m_2 a_1 a_2 (2 \dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_2^2) \sin \theta_2 \\
m_2 a_1 a_2 \dot{\theta}_1^2 \sin \theta_2 
\end{bmatrix} + \begin{bmatrix}
(m_1 + m_2) g a_1 \cos \theta_1 + m_2 g a_2 \cos (\theta_1 + \theta_2) \\
m_2 g a_2 \cos (\theta_1 + \theta_2)
\end{bmatrix} = \begin{bmatrix}
\tau_1 \\
\tau_2
\end{bmatrix}
\]

Where
The manipulator dynamics are in the standard form [2, 18]

\[ M(q) \ddot{q} + V(q, \dot{q}) + G(q) = \tau \]  

(2)

where \( M(q) \) is the \( n \times n \) symmetric positive definite manipulator inertia matrix, \( V(q, \dot{q}) \) is the \( n \times 1 \) vector of centrifugal and Coriolis torques, \( G(q) \) is the \( n \times 1 \) vector of gravitational torque, \( \tau \) is the \( n \times 1 \) vector of joint torque, \( q \) is the \( n \times 1 \) vector of the joint displacement (angular position), and \( \dot{q} \) and \( \ddot{q} \) are the \( n \times 1 \) vectors of the joint acceleration and velocity terms, respectively [2].

The units of elements of \( M(q) \) corresponding to revolute joint variables \( q_i = \theta_i \) are kg \( \cdot m^2 \). The units of the elements of \( M(q) \) corresponding to prismatic joint variables \( q_i = d_i \) are kilograms. The units of elements of \( V(q, \dot{q}) \) and \( G(q) \) corresponding to revolute joint variables are kg \( \cdot m^2 \)/s\(^2\). The units of elements of \( V(q, \dot{q}) \) and \( G(q) \) corresponding to prismatic joint variables are kg \( \cdot m \)/s\(^2\) [2].

Now the state-space formulations of the arm dynamics may be obtained by defining the position/velocity state \( x \in \mathbb{R}^{2n} \) as [2, p145]

\[ x = [q^T \dot{q}^T]^T \]

Equation (2) may be written as

\[ \frac{d}{dt} \dot{q} = -M^{-1}(q)[V(q, \dot{q}) + G(q)] + M^{-1}(q)\tau \]

(3)

Now, we may directly write the position/velocity state-space representation

\[ \dot{x} = \begin{bmatrix} 0 & I \\ -M^{-1}(q)N(q, \dot{q}) & 0 \end{bmatrix} \dot{q} + \begin{bmatrix} 0 \\ -M^{-1}(q) \end{bmatrix} \tau \]

(4)

Which is in the form of \( \dot{x} = f(x, u, t) \) with \( u(t) = \tau(t) \).

An alternative linear state equation of the form \( \dot{x} = Ax + Bu \) may be written as

\[ \dot{x} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ I \end{bmatrix} u \]

(5)

With control input defined by

\[ u(t) = -M^{-1}(q)N(q, \dot{q}) + M^{-1}(q)\tau \]

(6)

the control law

\[ \tau = N(q) - M(q)u + M(q)\ddot{q}_d \]

(7)

where

\[ u = \text{Control signal} \]

\[ \ddot{q}_d = \text{Desired trajectory} \]

Now in reality, a robot arm is always affected by friction and disturbances. Therefore, we shall generalize the arm model we have just derived by writing the manipulator dynamics as

\[ M(q) \ddot{q} + V(q, \dot{q}) + F(q) + G(q) + \tau_d = \tau \]

(8)

with \( q \) the joint variable \( n \)-vector and \( \tau \) the \( n \)-vector of generalized forces. \( M(q) \) is the inertia matrix, \( V(q, \dot{q}) \) the Coriolis/centripetal vector, and \( G(q) \) the gravity vector. We have added a friction term

\[ F(q) = F_v \dot{q} + F_d \]

(9)

With \( F_v \) the coefficient matrix of viscous friction and \( F_d \) a dynamic friction term. Also added is a disturbance \( \tau_d \), which could represent, for instance, any inaccurately modeled dynamics.

We shall sometimes write the arm dynamics as

\[ M(q) \ddot{q} + N(q, \dot{q}) + \tau_d = \tau \]

(10)

Where

\[ N(q, \dot{q}) \equiv V(q, \dot{q}) + F(q) + G(q) \]

(11)
Represent non linear terms.

**PROPERTIES OF ROBOTIC MANIPULATOR[2]**

Let us examine the structure and properties of each of the terms in the robot dynamics equation.

**Property 1:**
The inertia matrix \( M(q) \) is a positive definite symmetric matrix bounded by \( \mu_1 I \leq M(q) \leq \mu_2 I \), where \( \mu_1, \mu_2 \) are known positive constants[1,2,29].

\[
\begin{align*}
\mu_1 I & \leq M(q) \leq \mu_2 I \\
m_1 I & \leq M(q) \leq m_2 I
\end{align*}
\]  
(12) (13)

**Property 2:**
The matrix \( M(q) - 2V_m(q, \dot{q}) \) is skew-symmetric. This implies \( M(q) = V(q, \dot{q}) + V(q, \dot{q})^T \) (14)

\[
\begin{align*}
V(q, \dot{q}) & \text{ is quadratic in } \dot{q} \\
\|V(q, \dot{q})\| & \leq v_b \|\dot{q}\|^2
\end{align*}
\]  
(15)  
(16)

Where \( v_b(q) \) is a known scalar function, and for a revolute arm, \( v_b \) is a constant independent of \( q \). \( \|\cdot\| \) is any appropriate norm.

**Property 3:**
The viscous friction \( F_v \) may be assumed to have the form \( F_v = \text{diag}\{v_1\} \), with \( v_1 \) being known constant coefficients. The dynamic friction \( F_d \) may be assumed to have the form \( F_d(q, \dot{q}) = K_d \text{ sgn}(\dot{q}) \), with \( K_d = \text{diag}\{k_1\} \) being known constant coefficients. Thus, the bound on the friction terms may be assumed to be of the form

\[
\begin{align*}
F(q, \dot{q}) &= F_v(q) + F_d(q) \\
F_v &= \text{diag}\{v_1\} \\
F_d(q, \dot{q}) &= K_d \text{ sgn}(\dot{q}) \text{, with } K_d = \text{diag}\{k_1\} \\
\|F_v(q) + F_d(q, \dot{q})\| & \leq v \|\dot{q}\| + k
\end{align*}
\]  
(17) (18) (19) (20)

**Property 4:**
A bound on the gravity term may be derived for any given robotic manipulator

\[
\|G(q)\| \leq g_b
\]  
(21)

Where \( \|\cdot\| \) is any appropriate vector norm and \( g_b \) is a scalar function that may be determined for any given robotic manipulator.

**Property 5:**
The term \( \tau_d \) which could represent inaccurately modeled dynamics, and so on. We shall assume that it is bounded so that

\[
\|\tau_d\| \leq d,
\]  
(22)

Where \( d \) is a scalar constant that may be computed for a given arm and \( ||\cdot|| \) is any suitable norm.

**Property 6:**
The robot dynamical equation enjoys one last property that is linear in the parameters. This is important, since some or all of the parameters may be unknown; thus the dynamics are linear in the unknown terms [2,29]. This property may be expressed as

\[
M(q) \ddot{q} + V(q, \dot{q}) + F_v(q) + F_d(q, \dot{q}) + G(q) = M(q) \ddot{q} + N(q, \dot{q}) \equiv W(q, \dot{q}, \dot{q}) \phi
\]  
(23) (24)

with \( \phi \) the parameter vector and \( W(q, \dot{q}, \dot{q}) \) a matrix of robot functions depending on the joint variables, joint velocities, and joint accelerations. This matrix may be computed for any given robot arm and so is known. Note that the disturbance \( \tau_d \) is not included in this equation.
CONTROLLER DESIGN AND RESULTS

The following are the common parameters used in the simulation of all control laws.

<table>
<thead>
<tr>
<th>SYSTEM PARAMETER</th>
<th>LINK 1</th>
<th>LINK 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass(m) In kg</td>
<td>$m_1 = 1$</td>
<td>$m_2 = 1$</td>
</tr>
<tr>
<td>Length(l) In meter</td>
<td>$l_1 = 1$</td>
<td>$l_2 = 1$</td>
</tr>
<tr>
<td>Position or initial angle(q)</td>
<td>$\theta_1 = 0$°</td>
<td>$\theta_2 = 0$°</td>
</tr>
<tr>
<td>$\dot{\theta}_d$</td>
<td>$\theta_1 = 0$°</td>
<td>$\theta_2 = 0$°</td>
</tr>
</tbody>
</table>

Now the error is given as
\[ e = \begin{bmatrix} e_{11} \\ e_{21} \end{bmatrix}, \quad \dot{e} = \begin{bmatrix} \dot{e}_{11} \\ \dot{e}_{21} \end{bmatrix} \tag{25} \]

where
\[ e_{11} = \theta_{d1} - \theta_1, \tag{26} \]
\[ e_{21} = \theta_{d2} - \theta_2, \tag{27} \]
\[ \dot{e}_{11} = \dot{\theta}_{d1} - \dot{\theta}_1, \tag{28} \]
\[ \dot{e}_{21} = \dot{\theta}_{d2} - \dot{\theta}_2. \tag{29} \]

The torque is given by the equation as [2]
\[ \tau = N(q) - M(q)u + M(q)\ddot{\theta}_d \tag{30} \]
\[ \tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \text{ torque} \]
\[ \tau_1 = \text{Torque for link 1.} \]
\[ \tau_2 = \text{Torque for link 2.} \]
\[ u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \text{ Control signal.} \]
\[ u_1 = \text{Control signal for link 1.} \]
\[ u_2 = \text{Control signal for link 2.} \]

And acceleration ($\ddot{\theta}$) can be calculated by the equation given as [2,18]
\[ \ddot{\theta} = -M^{-1}(q)[N(q)] + M^{-1}(q)\tau \tag{31} \]

Now simulation for each control laws is given below one by one as

**Design of PD Controller**

The formula used for finding the control signal ‘u’ is given as
\[ u = K_pe(t) + K_d\dot{e}(t) \tag{32} \]

Where $K_p = 12$ and $K_d = 18$

Now the results for PD control of robotic 2 link arm are shown in Figs. 2 to 5.
Design of PID Controller

The formula used for finding the control signal (u) is given as

\[ u = K_p e(t) + K_i \int_0^t e(t) dt + K_d \dot{e}(t) \]  \hspace{1cm} (33)

where \( K_p = 50, K_i = 2 K_d = 60 \)

Now the results for PID control of robotic 2 link arm are shown in Figs. 6 to 9.
CONCLUSION

The performance of two-link robotic manipulator is investigated with PD, PID control. The PID controller resulted in the best performance and very effective and accurate trajectory tracking capability as compared to PD controller. Also the response with PID controller was having reduced oscillations about the desired trajectory as compared to PD controller.

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