Golden Research Thoughts
ISSN 2231-5063
Impact Factor : 3.4052(UIF)
Volume-4 | Issue-7 | Jan-2015
Available online at www.aygrt.isrj.org

ON ALMOST-CONTINUOUS MAPPINGS

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Abstract: In this paper, we introduce the concept of almost-continuous mappings. we give some characterizations of almost-continuous mappings by showing every continuous mapping is almost-continuous but the converse need not be true. Also we prove every almost-continuous mapping is weakly-continuous but the converse need not be true. But we prove an open mapping is almost-continuous if and only if it is weakly-continuous.

Keywords: Almost – continuous functions, continuous functions, almost-quasi-compact, regularly-open

INTRODUCTION

In this paper, we introduce the concept of almost-continuous mappings. we give some characterizations of almost-continuous mappings by showing every continuous mapping is almost-continuous but the converse need not be true. Also we prove every almost-continuous mapping is weakly-continuous but the converse need not be true. But we prove an open mapping is almost-continuous if and only if it is weakly-continuous.

Also we showed that composition of continuous function is almost-continuous is continuous. Also we discuss the product of almost-continuous and every restriction of an almost-continuous mapping is almost-continuous.

Definition:

A *topology* on a set is a collection τ of subsets of X having the following

properties:

(a) ϕ and X are in τ.

(b) The union of the elements of any sub collection of τ is in τ.

(c) The intersection of the elements of any finite subcollection of τ is in τ.

Definition:

Let (X,τ) be a topological space. A subset U of X is an *open* set of X if U belongs to the collection τ.

Example:

In the real line R, (a, b), (a, ∞), (b, ∞) are open.

Definition:

Let A be a subset of a topological space. A point x $\in$ A is said to be an *interior point* of A if A is a neighbourhood of x. The set of all interior points of A is called the interior of A .

We write A° or Int A for the interior of A. A is open if and only if $A = A^\circ$.

Lemma:

Let A and B be a subset of X. Then

(1) $X^\circ = X$ and $\phi^\circ = \phi$.
(2) $A^\circ \subset A$.
(3) $(A^\circ)^\circ = A$.
(4) $A \subset B \Rightarrow A^\circ \subset B^\circ$.
(5) $(A \cap B)^\circ = A^\circ \cap B^\circ$ and $A^\circ \cup B^\circ \subset (A \cup B)^\circ$.

Definition:

Let (X,τ) be a topological space. A subset U of X is said to be*closed*iftheset X-U is open.

Example:

The subset [a, b] of R is closed.
R – [a, b] = (-, ∞, a) $\cup$ (b, ∞).
But (-, ∞, a) and (b,, ∞) are open.
Therefore, R-[a, b] is open.
Therefore, [a, b] is closed.

Definition:

A mapping f:X→Y is said to be *almost-continuous*[5] at a point x $\in$ X, if for every neighbourhood M of f(x), there is a neighbourhood N of x such that $f(N) \subset \bar{M}^\circ$

Theorem:

Every continuous mapping is almost-continuous.

Proof:

Let f : (X, τ) $\longrightarrow$ (Y, τ*) be a continuous mappings.
Let x $\in$ X.
Then f(x) $\in$ Y and M is a neighbourhood of f(x) in Y.
Then there exists a neighbourhood N of x such that f (N) $\subset$ M.
Since f is continuous and M is open, $\bar{M}^\circ$ is also open.
Therefore, $f(N) \subset M = \bar{M}^\circ$.

Hence f (N) $\subset$ M $^{-\circ}$.
Hence f is almost-continuous. ■
The converse of the above Theorem need not be true in general as shown by the following Example.

Example:

Let R be the set of real numbers and
$\tau = \{\phi, R\} \bigcup \{U \subset X: X - U$ is countable or all of X$\}$. Let X = {a, b} and
let $\tau^* = \{\phi, \{a\}, X\}$. Let f: $(R, \tau) \rightarrow (R, \tau^*)$ be defined by

$$f(x) = \begin{cases} a \text{ if } x \text{ is rational} \\ b \text{ if } x \text{ is irrational} \end{cases}$$

Then f is continuous at each point of R, but f is not continuous at $x \in R$ if x is rational.

Proof:

Let $x \in Q$.
Then f(x) = {a}. Open sets containing a are {a, b} and X.
It is enough to check for {a}.
Let U be a neighbourhood of {a}.
Then $U^{-\circ} = X$.
Now choose any open set V containing x, it must contain both Q and Q^c.
$f(V) = \{a, b\} \subseteq U^{-\circ} = \{a\}^{-\circ}$.
Hence f is almost-continuous at Q.
Let $x \in Q^c$.
Then f(x) = {b}. Open set containing {b} is X.
Let U be a neighbourhood of {b}.
Then $U^{-\circ} = X$.
Now choose an open set V containing x.
Therefore, $f(V) \subseteq U^{-\circ}$.
Therefore, f is almost-continuous at $x \in Q^c$.
Hence f is almost-continuous .
Let $x \in Q$.
Then f(x) = a and $f(x) \in V$.
Now $a \in V = \{a\}$.
Then $f^{-1}(\{a\}) = Q$.
But Q is not open in τ^*.
Therefore, f is not continuous at $x \in Q$.

Definition:

A mapping $f:X \rightarrow Y$ is said to be *weakly-continuous* [5] if for each point $x \in X$ and each neighbourhood V of f(x), there exists a neighbourhood U of x such that $f(U) \subset \overline{V}$.

Theorem:

Every almost-continuous mapping is weakly continuous.

Proof.

Let f be an almost-continuous mapping.

Claim: f is weakly continuous.

Let $x \in X$.
Then $f(x) \in Y$ and M is a neighbourhood of f(x).
Since f is almost-continuous, there exists a neighbourhood N of x such that $f(N) \subseteq M^{-\circ}$.
But M is a regularly open neighbourhood of f(x).
Therefore, $f(N) \subseteq M^{-\circ} = M^{-}$ where M^{-} is a open neighbourhood.
Therefore, $f(N) \subseteq M^{-}$.
Hence f is weakly-continuous. ■
The following Example shows that the converse of the above Theorem need not be true.

Example:

Let (R,τ) be the space as in above Example . Let X={a, b, c} and
let $\tau^* = \{\phi, \{a\}, \{c\}, \{a,c\}, X\}$.
Let f be a mapping of (R, τ) into (X,τ^*) defined as follows:

$$f(x) = \begin{cases} a & \text{if x is rational} \\ b & \text{if x is irrational} \end{cases}$$

Then f is a weakly-continuous open mapping which is not almost-continuous
(at any rational point).

Proof:

Let $x \in Q$.
Then $f(x) = \{a\} \in Q$.
Then $f(x) \in U$ where U is a neighbourhood of f(x) and it must contain {a, b}.
Therefore, there exists a neighbourhood V of x such that f (V) = {a, b}.
That is, $f(V) \subseteq U^{-}$.
Therefore, f is weakly-continuous.
Let $x \in Q$.
Then f(x) = {a}, {a} is an open set.
Then $\overline{\{a\}} = \{a, b\}$
That is, $\{a\}^{-\circ} = \{a\}$.
That is, $x \subseteq U$, U must contain Q and Q^c.
Therefore, $f(U) = \{a, b\} \not\subset \{a\}^{-\circ} = \{a\}$.
Therefore, f is not almost-continuous.

Definition:

A mapping $f : X \rightarrow Y$ is said to be *almost-quasi-compact* [5] if it is onto and if A is open whenever $f^{-1}(A)$ is regularly-open.

Theorem:

Suppose that f maps X onto Y and g maps Y onto Z. Then if f is almost-continuous and $g \circ f$ is open then g is almost-open.

Proof:

Suppose that f is almost-continuous and $g \circ f$ is open.
Let S be any regularly-open subset of Y.
Since f is almost-continuous, then $f^{-1}(S)$ is an open subset of X.
Now, $g \circ f$ is open.
Therefore, $(g \circ f)(f^{-1}(S))$ is also open.
But $(g \circ f)(f^{-1}(S)) = g(S)$.
Therefore, g(S) is open.
Therefore, g is almost-open. ■

Theorem:

Suppose that f maps X onto Y and g maps Y onto Z. Then if f is almost-continuous and if $g \circ f$ is closed then g is almost-closed.
Proof.
Suppose that f is almost-continuous $g \circ f$ is closed.
Claim: g is almost-closed.
Let S be any regularly-closed subset of Y.
Since f is almost-continuous, $f^{-1}(S)$ is a closed subset of X.
Now, $g \circ f$ is closed.
Therefore, $(g \circ f)(f^{-1}(S))$ is also closed.
But $(g \circ f)(f^{-1}(S)) = g(S)$.
Therefore, g(S) is closed.
Therefore, g is almost-closed. ■

Theorem:

Suppose that f maps X onto Y and g maps Y onto Z. Then if f is almost-continuous and if $g \circ f$ is quasi-compact then g is almost-quasi-compact.

Proof:

Suppose that f is almost-continuous and $g \circ f$ is quasi-compact.
Let $g^{-1}(S)$ be a regularly-open subset of Y.
Then, by almost-continuity of f, $f^{-1}(g^{-1}(S))$ is open
But $f^{-1}(g^{-1}(S)) = (g \circ f)^{-1}(S)$.
Since $g \circ f$ is quasi-compact, S must be open.
Therefore, g is almost-quasi-compact. ■

CONCLUSION

In this Paper, we have proved that every continuous mappings is almost-continuous mappings but the converse need not be true. We have also proved that every weakly-continuous mappings is almost-continuous but the converse need not true.

REFERENCES

1. Chandrasekhara Rao K., Topology, Narosa Publishing House Private Limited New Delhi.
2. James R. Munkres, Topology, Prentice Hall of India Private Limited.,New Delhi, 11001.
3. Mangesh G. Murdeshwar, General Topology, Published by Mohinder Singh Sejwal for Wiley Eastern Limited, New Delhi, 1986 .
4. Raisinghania M.D. and R.S. Agarwal, Topology, S.Chand and Company Limited, Ram Nagar, New Delhi-11055, 1987.
5. Singal M.K. and Asha Rani Singal(1968), Almost Continuous Mappings,Yokohama Math. J., 16, 63-73.