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SOLVING FUZZY ASSIGNMENT PROBLEM WITH TRAPEZOIDAL FUZZY NUMBERS USING ROBUST RANKING AND ZERO SUFFIX METHOD

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Abstract:-In this paper, fuzzy assignment problem is solved by using Robust Ranking and Zero Suffix method in which the assignment costs are trapezoidal fuzzy numbers. In this method fuzzy assignment problem has been first transformed into crisp assignment problem using Robust's ranking method and then solve it by using zero suffix method. This method requires least iterations to reach optimality as compaired to the existing methods available in the literature. Here a numerical example is solved to check the validity of the proposed method.

Keywords: Assignment problem, trapezoidal fuzzy numbers, Robust Ranking method, Zero Suffix method

INTRODUCTION

In real world problems, we are faced with the problem of allocating different persons/ workers/machines to different jobs. Not everyone has the same ability to perform a given job. Different persons have different abilities to execute the same task and these different capabilities are expressed in terms of cost/profit/time involved in executing a given job. Therefore, we have to decide how to assign different workers to different jobs" so that, cost of performing such job is minimized. In this paper, we investigate more realistic problems, namely the assignment problem with fuzzy costs ãij. Since the objectives are to minimize the total cost or to maximize the total profit, subject to some crisp constraints, the objective function is considered also as a fuzzy number. The method is to rank the fuzzy objective values of the objective function by some ranking method for fuzzy numbers to find the best alternative. On the basis of this idea the Robust's ranking method [13] has been adopted to transform the fuzzy assignment problem to a crisp one so that the conventional solution methods may be applied to solve assignment problem. The idea is to transform a problem with fuzzy parameters to a crisp version in the LPP form and solve it by the zero suffix method. Other than the fuzzy assignment problem other applications of this method can be tried in project scheduling, maximal flow, transportation problem, job-shop scheduling etc. Barr at al.[2] presents Cost minimizing assignment problems and Ravindran et al. [8] presentsTime minimizing assignment problems. Fuzzy numbers can be explained by many ranking methods in [3,5,13]. Rupsha Roy et.al [9] proposed a method to solve assignment problem by using zero suffix method. Nagarajan R. et.al [7] has been solved fuzzy assignment problem by ranking method. Different kinds of transportation problems are solved by different methods in the articles[1,4,6,10,11,12]Zadeh [14] in 1965 first introduced Fuzzy set as a mathematical way of representing impreciseness or vagueness in everyday life.

2. BASIC DEFINITIONS

2.1. Fuzzy set: A fuzzy set is characterized by a membership function mapping element of a domain, space or universe of discourse X to the unit interval [0, 1] i.e. A = {(x, μA(x); x ∈X}, Here μA: X → [0, 1] is a mapping called the degree of membership function of the fuzzy set A and μA(x) is called the membership value of x∈X in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0,1].
2.2. Trapezoidal fuzzy numbers : A fuzzy number Ã= (a1,a2,a3,a4) is said to be trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1} & \text{if } a_1 \le x \le a_2\\ 1 & \text{if } a_2 \le x \le a_3\\ \dfrac{a_4-x}{a_4-a_3} & \text{if } a_3 \le x \le a_4\\ 0 & \text{otherwise}\end{cases}$$

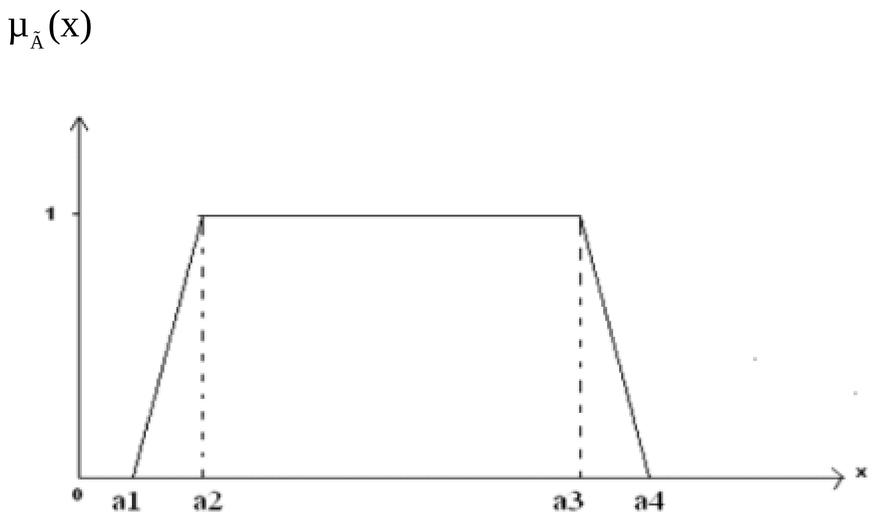

Fig-1.6- Trapezoidal Membership Function

3. FUZZY ASSIGNMENT PROBLEM

Fuzzy assignment problem can defined as following mathematical form
Minimize

$$Minimize\ \tilde{Z} = \sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{a}_{ij} X_{ij}$$

$$\sum_{i=1}^{n} X_{ij} = 1, \qquad j=1,2,\ldots,n$$

$$\sum_{j=1}^{n} X_{ij} = 1, \qquad i=1,2,\ldots,n$$

$$x_{ij} = \begin{cases}1, \text{ if the } i^{th} \text{ job is assigned to the } j^{th} \text{ machine}\\ 0, \text{ if the } i^{th} \text{ job is not assigned to the } j^{th} \text{ machine}\end{cases}$$

4. ROBUST RANKING TECHNIQUE:

Roubast ranking technique which satisfy compensation, linearity, and additivity properties and provides results which are consist human intuition. If ã is a fuzzy number then the Roubast Ranking is defined by $R(\tilde{a}) = \int_0^1 (0.5)(a_\alpha^L, a_\alpha^U)\, d\alpha$, where (a_α^L, a_α^U) is the a level cut of the fuzzy number ã

$R(\tilde{a}) = \int_0^1 (0.5)(a_\alpha^L, a_\alpha^U)\, d\alpha$, where (a_α^L, a_α^U) is the α-level cut of the fuzzy number ã

5 ZERO SUFFIX METHOD

The working rule of finding the optimal solution is as follows:
Step 1: Construct the assignment problem.
Step2: Subtract each row entries of the assignment table from the row minimum element.
Step 3: Subtract each column entries of the assignment table from the column minimum element.
Step 4: In the reduced cost matrix there will be at least one zero in each row and column, then find the suffix value of all the zeros in the reduced cost matrix by following simplification, the suffix value is denoted by S.
Therefore,

S= Add the cost of nearest adjacent sides of zeros/No. of costs added

Step 5: Choose the maximum of S, if it has one maximum value then assign that task to the person and if it has more than one maximum value then also assign the tasks to their respective persons (if the zeros don't lie in the same column or row).

And if the zeros lie in the same row or column then assign the job to that person whose cost is minimum. Now create a new assignment table by deleting that row & column which has been assigned.
Step 6: Repeat step 2 to step 3 until all the tasks has not been assigned to the persons.

6. SOLUTION ALGORITHM

Step 1: Construct the fuzzy assignment problem
Step 2: Convert the fuzzy assignment problem into crisp assignment problem by using Robust's ranking technique.
Step 3: Apply the zero suffix method to get optimal solution.

7. NUMERICAL EXAMPLE

Let us consider a Fuzzy Assignment Problem with rows representing 5 persons P_1, P_2, P_3, P_4, P_5 and columns representing the 5 jobs J_1, J_2, J_3, J_4, J_5. The cost matrix [ãij] is given whose elements are ***Trapezoidal fuzzy numbers.*** The problem is to find the optimal assignment so that the total cost of job assignment becomes minimum.

$[\tilde{a}_{ij}]=$

(4,6,7,9)	(3,5,7,10)	(5,7,10,12)	(3,4,6,9)	(4,5,7,10)
(2,3,5,9)	(5,7,9,13)	(4,6,9,12)	(5,6,7,10)	(2,3,5,7)
(7,9,10,12)	(6,7,9,10)	(7,9,10,13)	(6,7,10,13)	(7,10,13,14)
(4,5,7,9)	(5,7,12,15)	(7,9,13,15)	(2,4,10,13)	(5,7,10,14)
(4,10,13,15)	(3,7,9,13)	(2,3,10,14)	(3,7,10,13)	(4,7,10,14)

Solution: The fuzzy Assignment problem is given as

	J_1	J_2	J_3	J_4	J_5
P_1	(4,6,7,9)	(3,5,7,10)	(5,7,10,12)	(3,4,6,9)	(4,5,7,10)
P_2	(2,3,5,9)	(5,7,9,13)	(4,6,9,12)	(5,6,7,10)	(2,3,5,7)
P_3	(7,9,10,12)	(6,7,9,10)	(7,9,10,13)	(6,7,10,13)	(7,10,13,14)
P_4	(4,5,7,9)	(5,7,12,15)	(7,9,13,15)	(2,4,10,13)	(5,7,10,14)
P_5	(4,10,13,15)	(3,7,9,13)	(2,3,10,14)	(3,7,10,13)	(4,7,10,14)

In Conformation to model the fuzzy assignment problem can be formulated in the following mathematical programming form

Min{R(4,6,7,9)x_{11}+R(3,5,7,10)x_{12}+R(5,7,10,12)x_{13}+R(3,4,6,9)x_{14}+R(4,5,7,10)x_{15} +R(2,3,5,9)x_{21}+R(5,7,9,13)x_{22}+R(4,6,9,12)x_{23}+R(5,6,7,10)x_{24}+R(2,3,5,7)x_{25}+R(7,9,10,12)x_{31}+R(6,7,9,10)x_{32}+R(7,9, 10,13)x_{33}+R(6,7,10,13)x_{34}+R(7,10,13,14)x_{35}+R(4,5,7,9)x_{41}+R(5,7,12,15)x_{42}+R(7,9,13,15)x_{43}+R(2,4,10,13)x_{44}+R(5,7 ,10,14)x_{45}+R(4,10,13,15)x_{51}+R(3,7,9,13)x_{52}+R(2,3,10,14)x_{53}+R(3,7,10,13)x_{54}+R(4,7,10,14)x_{55}}
Subject to

Subject to

$x_{11}+x_{12}+x_{13}+x_{14}=1;\ x_{11}+x_{21}+x_{31}+x_{41}=1$
$x_{21}+x_{22}+x_{23}+x_{24}=1;\ x_{12}+x_{22}+x_{32}+x_{42}=1$
$x_{31}+x_{32}+x_{33}+x_{34}=1;\ x_{13}+x_{23}+x_{33}+x_{43}=1$
$x_{41}+x_{42}+x_{43}+x_{44}=1;\ x_{14}+x_{24}+x_{34}+x_{44}=1$ xij Є [0,1]

Now we calculate R(4,6,7,9) by applying Robust's ranking method.

$$R(\tilde{a}_{ij}) = R(4,6,7,9) = \int_0^1 (0.5)(a^L_\alpha, a^U_\alpha)\,d\alpha,$$

\ $$Where\ (a^L_\alpha, a^U_\alpha) = \{(a_2 - a_1)\alpha + a_1, a_4 - (a_4 - a_3)\alpha\}$$
$$(a^L_\alpha, a^U_\alpha) = (2\alpha + 4, 9 - 2\alpha) = 13$$

$$R(\tilde{a}_{ij}) = R(4,6,7,9) = \int_0^1 (0.5)(13)\,d\alpha = 6.5$$

Proceeding similarly, the Robust's ranking indices for the fuzzy costs are calculated as:

R(3,5,7,10) = 6.25, R(5,7,10,12) = 8.5, R(3,4,6,9) = 5.5, R(4,5,7,10) = 6.5, R(2,3,5,9) = 4.75, R(5,7,9,13) = 8.5, R(4,6,9,12) = 15.5, R(5,6,7,10) = 7, R(2,3,5,7)= 4.25, R(7,9,10,12) =9.5, R(6,7,9,10) = 8, R(7,9,10,13) = 9.75, R(6,7,10,13) = 9, R(7,10,13,14) = 11, R(4,5,7,9) = 6.25, R(5,7,12,15) = 9.75, R(7,9,13,15) =11, R(2,4,10,13)= 7.25, R(5,7,10,14) = 9, R(4,10,13,15) = 10.5, R(3,7,9,13) = 8, R(2,3,10,14) = 7.25, R(3,7,10,13) =7.75, R(4,7,10,14) = 8.75.

After Ranking the assignment problem will be as

	J_1	J_2	J_3	J_4	J_5
P_1	6.5	6.25	8.5	5.5	6.5
P_2	4.75	8.5	15.5	7	4.25
P_3	9.5	8	9.75	9	11
P_4	6.25	9.75	11	7.25	9
P_5	10.5	8	7.25	7.75	8.75

By applying the zero suffix method we get the following allocation table

\

	J_1	J_2	J_3	J_4	J_5
P_1	6.5	6.25	8.5	**5.5***	6.5
P_2	4.75	8.5	15.5	7	**4.25***
P_3	9.5	**8***	9.75	9	11
P_4	**6.25***	9.75	11	7.25	9
P_5	10.5	8	**7.25***	7.75	8.75

*optimal assignment

And the optimal solution is

x*14 = x*25 = x*32=x*41 = x*53 = 1

x*11=x*12=x*13=x*15=x*21=x*22=x*23=x*24=x*31=x*33=x*34=

x*35=x*42=x*43=x*44=x*45=x*51=x*52=x*54=x*55=0 with

optimal objective value R(z Þ*) = 31.25 which represents a optimal total cost. In other words the optimal assignment is

Persons→Job		Cost
P_1	→J_4	5.5
P_2	→J_5	4.25
P_3	→J_2	8
P_4	→J_1	6.25
P_5	→J_3	7.25

The fuzzy allocation table is

	J_1	J_2	J_3	J_4	J_5
P_1	(4,6,7,9)	(3,5,7,10)	(5,7,10,12)	**(3,4,6,9)***	(4,5,7,10)
P_2	(2,3,5,9)	(5,7,9,13)	(4,6,9,12)	(5,6,7,10)	**(2,3,5,7)***
P_3	(7,9,10,12)	**(6,7,9,10)***	(7,9,10,13)	(6,7,10,13)	(7,10,13,14)
P_4	**(4,5,7,9)***	(5,7,12,15)	(7,9,13,15)	(2,4,10,13)	(5,7,10,14)
P_5	(4,10,13,15)	(3,7,9,13)	**(2,3,10,14)***	(3,7,10,13)	(4,7,10,14)

*fuzzy optimal assignment

And the fuzzy optimal total cost is

ã14 + ã25 + ã32+ ã41 + ã53 = R(3,4,6,9)+ R(2,3,5,7) +R(6,7,9,10)+R(4,5,7,9) +R(2,3,10,14) =R(17,22,37,49) .

Also we find that R(z Ã*) = 31.25.

In this example, it has Seen that the total optimal cost obtained by our methods remains as that obtained by defuzzifying the total fuzzy optimal cost by applying the Robust's ranking method and usual Hungarian method.

8. CONCLUSIONS

In this paper, the assignment costs are considered as imprecise numbers described by fuzzy numbers which are more realistic and general in nature. Moreover, the fuzzy assignment problem has been first transformed into crisp assignment problem using Robust's ranking method and then solve it by using zero suffix method. Numerical examples show that by this method we can have the optimal assignment as well as the crisp and fuzzy optimal total cost. Moreover, one can conclude that the solution of fuzzy problems can be obtained by Robust's ranking method and zero suffix method effectively. This technique can also be used in solving other types of problems like, project schedules, job-shop scheduling problems, transportation problems and network flow problems.

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