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ORIGINAL ARTICLE

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APPLICATIONS OF TOPOLOGY IN REAL LIFE

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Abstract:

Scientists and engineers work with real and complex numbers. In a quantity of cases, they utilize topological properties of these numbers. To find derivatives and integrals, we need existence of limits. In their turn, limits are topological constructions. It means that manipulations with limits and, consequently, with derivatives and integrals are based on the topology of number spaces. Number spaces, the real and complex lines, as well as Euclidean spaces have a good topology that allows mathematicians to develop calculus and optimization methods in these spaces. They are metric spaces, which possess a lot of useful features. These features provide for solution of many theoretical and practical problems.

KEY WORDS:

Topology; relative continuity; discontinuous structure; relative connectedness; fuzzy limit.

INTRODUCTION

In mathematics, topology, the study of topological spaces, is an area of mathematics concerned with the properties of space that are preserved under continuous deformations, such as stretching and bending, but not tearing or gluing. Important topological properties include connectedness and compactness.

Topology developed as a field of study out of geometry and set theory, through analysis of such concepts as space, dimension, and transformation. Such ideas go back to Gottfried Leibniz, who in the 17th century envisioned the geometria situs (Greek-Latin for "geometry of place") and analysis situs (Greek-Latin for "picking apart of place"). The term topology was introduced by Johann Benedict Listing in the 19th century, although it was not until the first decades of the 20th century that the idea of a topological space was developed. By the middle of the 20th century, topology had become a major branch of mathematics.

TOPOLOGY HAS MANY SUBFIELDS:

•General topology establishes the foundational aspects of topology and investigates properties of topological spaces and investigates concepts inherent to topological spaces. It includes point-set topology, which is the foundational topology used in all other branches (including topics like compactness and connectedness).
•Algebraic topology tries to measure degrees of connectivity using algebraic constructs such as homology and homotopy groups.
•Differential topology is the field dealing with differentiable functions on differentiable manifolds. It is closely related to differential geometry and together they make up the geometric theory of differentiable

manifolds.
•Geometric topology primarily studies manifolds and their embeddings (placements) in other manifolds. A particularly active area is low dimensional topology, which studies manifolds of four or fewer dimensions. This includes knot theory, the study of mathematical knots.
•Topology began with the investigation of certain questions in geometry. Leonhard Euler's 1736 paper on the Seven Bridges of Königsberg[1] is regarded as one of the first academic treatises in modern topology.
•The term "Topologie" was introduced in German in 1847 by Johann Benedict Listing in Vorstudien zur Topologie,[2] who had used the word for ten years in correspondence before its first appearance in print. The English form topology was first used in 1883 in Listing's obituary in the journal Nature[3] to distinguish "qualitative geometry from the ordinary geometry in which quantitative relations chiefly are treated". The term topologist in the sense of a specialist in topology was used in 1905 in the magazine Spectator.[citation needed] However, none of these uses corresponds exactly to the modern definition of topology.
•Modern topology depends strongly on the ideas of set theory, developed by Georg Cantor in the later part of the 19th century. In addition to establishing the basic ideas of set theory, Cantor considered point sets in Euclidean space as part of his study of Fourier series.
•Henri Poincaré published Analysis Situs in 1895,[4] introducing the concepts of homotopy and homology, which are now considered part of algebraic topology.
•Unifying the work on function spaces of Georg Cantor, Vito Volterra, Cesare Arzelà, Jacques Hadamard, Giulio Ascoli and others, Maurice Fréchet introduced the metric space in 1906.[5] A metric space is now considered a special case of a general topological space. In 1914, Felix Hausdorff coined the term "topological space" and gave the definition for what is now called a Hausdorff space.[6] Currently, a topological space is a slight generalization of Hausdorff spaces, given in 1922 by Kazimierz Kuratowski
•For further developments, see point-set topology and algebraic topology.

Think of geometric shapes like a circle, a square, and an annulus (the region between two concentric circles) geometrically, they are very different. But there is something the circle and square have in common, a property not shared by the annulus. Start at a point inside the circle or square and draw any loop that remains in the circle or square and returns to the starting point. That loop will have the property that you can "shrink" it down to a point without any portion of the loop ever leaving the circle or square.

But in the annulus, it is possible to draw a loop that surrounds the "hole" formed by the inner circle. It will be impossible to shrink that loop to a point without dragging part of it into the hole, and thus leaving the annulus.

Think about which geometric regions have this shrink-to-a-point property and which do not. You'll soon discover that the distinction between the two types of regions has nothing to do with geometric properties like angles, areas, distances, etc. We call this loop property a topological property.

So, loosely, the subject of topology studies a region by looking at the properties of that region that are independent of the geometric properties of the region.

but it gets more abstract than that. Typically, the regions being studied aren't regions in the plane, except for textbook examples. They tend to be regions located in "topological spaces", which are very abstract, often multidimensional, even infinite dimensional!

As for applications, I think it is fair to say that most of the applications of topology are not directly to "real life." On the other hand, topology provides mathematical tools that are useful to applied mathematicians and to theoretical physicists when they do their work.

There is a theory among physicists that the universe is not just 3 dimensional, but that there are extra dimensions curled up in a complicated way on an infinitesimally tiny scale. Topology is used to describe this weird higher dimensional space we might be living in.

Topology is the study of which shapes are "topologically similar".

Very generally speaking: Two shapes are "topologically similar" if you can match them by "stretching smoothly" without bunching or overlapping (called a 1-to-1 mapping) and whose boundaries (edges) are also the same shape (topologically).

1. Take a disk - a filled circle. In topology, a disk is the same as (topologically similar to) a "birthday hat" (an empty cone with no bottom).

Why: First notice that edge/boundary of both the disk and the birthday hat are circles. Now think of the disk as a circle cut out of a balloon so that it is stretchy. If you are careful, you can glue it onto the birthday hat without overlapping or creasing by putting the center at the top point and pulling the boundary of the circle along the base of the hat. In this way each point of the balloon disk corresponds to a unique point on the birthday hat and vice-versa.

2.A disk is also topologically similar to a (filled) square or rectangle or triangle or indeed any shape cut out of cardboard with a single non-crossing, non-touching boundary, because angles do not "count" in topology. Again, one can stretch and glue the balloon disk onto a piece of cardboard cut into the shape.
3.A disk is NOT topologically similar to a (filled) figure 8 because there is no way to glue the balloon disk onto the figure 8 without "bunching" or overlapping of points around the crossing point.
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Topology also looks at "direction" or orient ability.

1.Cut out two strips of paper. Draw a red line down the length of one side of each strip and a blue line down the other side. Mark arrows along the lines |----->----->---->----->------>-----|.
Now glue the ends of the first strip together to make a cylinder (red to red) and glue the ends of the other strip together to make a mobius band (red to blue).
1.A cylinder is NOT topologically similar to a mobius band since a cylinder has two sides and a mobius band only one. There is no way to match them up.
2.A mobius band is "interesting" in topology because it is not "orientable" since as you go along the line it goes from --->----- to -----<------ to ---->----- . It constantly changes direction or "orientation".
3. Notice that the cylinder band is orientable since as you go along a line on the cylinder the arrows always are in the same direction.
3.A topologist can't tell the difference between a coffee cup and a donut, or so the saying goes. That's because topology is the study of geometrical objects without considering things like length, angles, or shape. A coffee cup can be transformed into a donut by squashing, stretching and twisting, so the two objects are topologically equivalent. However, a ball can't be turned into a donut without tearing or cutting, so they are topologically different.
It makes for some clever party tricks, but does topology have any practical applications? If you've ever looked at a map of the London Underground, you've seen topology in action. This iconic representation of the Tube ignores the distances and physical locations of the various stations, but preserves the links between them. The resulting map is much clearer than the unwieldy real life mess.

Topology also comes in handy elsewhere. Cosmologists use a lot of topology when they are studying the structure of our universe. The exact shape of our universe has very important implications for how it began, how it behaves today, and how it might end. Researchers believe the universe could be in the shape of a sphere, a saddle, or even a horn.

Back on Earth, engineers put topology to work helping them design robots. The movements that robots can make can be thought of as shapes in n-dimensional space, where n is the number of joints the robot has. By exploring the shapes with topology, roboticists can get a better idea of how their robots will move.

Topology is often associated with highly theoretical maths, but these examples show it does have some practical uses. Whichever career path you are interested in, have a look at these links for more information:

Physical scientists mostly use only the mathematical formalisms of topology, applying them to computer modelling, using to point-set theory and systemic concepts. Note that there is a fundamental but simple difference between mathematical topology and the basic form of geometrictopology used here.

This work describes an 'imaging' method called 'Nexial-Topology' that is purely geometric. It does make use of computer animations created by topologists, for the purpose of visual imaging, but not of the measures or counts associated with their programming, design, or calculation. Topology is used here to 'picture a likeness' of the generic notion of 'deployment', not to figure any real event or physical object or space.

This topology is also presented as a 'native gauging' ability (in an 'undeployed' form), which is an application of basic topologic properties of distortion — that is, without holes or the discontinuities characteristic of deployed perspectives, systems of analysis, and scientific topologies. These are used in highly specialized and technical fields that are outside the domain of application of this work. This generalist work concerns generic situation modeling.

The difference may appear obscure (see note 4 in Introduction about this term), so an idiosyncratic history of the development of topology is included in appendix, but the best way to apprehend this distinction is to perform the two cognitive experiments included, and peruse the animations within the context of daily life and general philosophies, while reading this book. To summarize, 'nexial' topology is the essential nature of the topologies that mathematics developed (describing various ways of creating a 'nexus' that represents 'reality'), and the 'native gauging' is a non-nexial form of topology from which all our other representations arise.

Human scientists and thinkers are not familiar with topology, and its geometric way of representing situations is alien to most. The appendix about topology discusses its history, shifting meanings, and gives some familiar pointers. There are three ways to approach the nature of this geometric discipline, and to gain an idea of what is involved in this work.

If 'deployed', the geometric aspect of topology is directly related to flat images in 'symbolic thought': the symbols used in philosophies, theories, and general models, are like instantaneous pictures of a global deployment of what comes to 'exist', a description of what is 'extended' (think of 'expansion') or localised in one reality or another. This is a 'thinking in image'.

If undeployed, it is a 'native gauging' ability that is related to non-sensory sensation and to unconscious gesture that some of us often use to express situations and reactions we relate to instinct or intuition or gut feeling, to stress or being unaffected, or to express what we consider 'spontaneous that is not reactive or powerful.

The most direct way to get a sense of what this means is to perform the two cognitive experiments provided, peruse the animations and, while reading this book, think of personal experience, sensation, and gesture.

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