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ORIGINAL ARTICLE

A PURELY SEQUENTIAL PROCEDURE FOR FIXED WIDTH CONFIDENCE INTERVAL FOR THE PARAMETER OF A DENSITY WHO'S LIMITS DEPEND UPON THE PARAMETER

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ABSTRACT: In the literature, an extensive work on sequential fixed width confidence interval (CI) for the parameter of U (0, θ) model is available. Here the upper limit depends on the parameter. In this article we propose fixed width (1-α) level sequential CI for the parameter of a density whose both limits depend upon the parameter.

Keywords: Coverage probability; Uniform density; Average sample number (ASN).

Mathematics Subject Classification: Primary 62L12; Secondary 62F25.

1. INTRODUCTION

Sequential estimation procedures for the parameter of U(0, θ) distribution have been studied by many authors, for examples Graybill and Connell (1964), Cooke (1971, 1973), Govindarajulu (1997,1999). Akahira and Koike (2005) have considered the problem of finding (1-α) level fixed width confidence interval (CI) for θ in U($\theta-\xi/2$, $\theta+\xi/2$) distribution, where ξ is unknown and independent of θ. Recently Koike (2007) has generalized this for a location-scale family of distribution with a finite support on the interval ($\theta-\xi$a, $\theta+\xi$a), 'a' is finite known positive number. For further details on sequential estimation one may refer to Ghosh et al.(1997).

In this article we propose sequential procedure for (1-α) level fixed width CI for the parameter θ of a density whose limits depend upon θ and are strictly increasing continuous functions of θ. Similar results can be obtained when both the limits are strictly decreasing continuous functions of θ. However these conditions can be relaxed to some extent (see remark 3.2).

Section 2 contains some related preliminary results. In Sections 3 we propose a purely sequential procedure. In Section 4 as an illustration, for U(θ, mθ) distribution with m known, by simulation it is verified that the desired level is attained and also as expected, the ASN increases when θ and/or m increase.

2. PRELIMINARY RESULTS:

Let the probability density function (p.d.f) of a random variable X be

$$f(x,\theta) = \begin{cases} g(x)/h(\theta), & \text{for } a(\theta) < x < b(\theta);\ \theta \in \Theta \subset R, \\ 0, & \text{otherwise,} \end{cases} \qquad (2.1)$$

where $g(x) \geq 0$ be a known function, $h(\theta) = \int_{a(\theta)}^{b(\theta)} g(x)\, dx$ be positive and finite. In the following for simplicity we assume that the known functions $a(\theta)$, $b(\theta)$ are strictly increasing and continuous.

The model (2.1) is appropriate in a situation similar to the following. Consider an agricultural experiment where we want to study the impact of unknown soil fertility gradient θ of a plot, on the yield/growth of a certain crop which is an observable random variable, say X, whose range depend on θ, say $a(\theta)$ and $b(\theta)$. It is but natural to assume that both $a(\theta)$ and $b(\theta)$ are continuous strictly increasing functions of θ. Being assumed that θ is the only unknown entity, the random variable X can be modeled by (2.1). The problem of interest is to find a fixed width $(1-\alpha)$ level CI for the parameter θ.

Let $\underline{\theta} = \inf\{\theta\in\Theta\}$, $\overline{\theta} = \sup\{\theta\in\Theta\}$ and $\underline{a} = \inf\{a(\theta): \theta \in\Theta\}$, $\overline{a} = \sup\{a(\theta) : \theta\in\Theta\}$. Similarly $\underline{b}$, $\overline{b}$ are defined. It is to be noted that the sample space is a subset of $(\underline{a}, \overline{b})$, whatsoever be $\theta \in (\underline{\theta}, \overline{\theta})$. Hence for $t \in (\underline{a}, \overline{b})$ we define inverse function $a^{-1}(t) = \underline{\theta}$ or $\overline{\theta}$ according as $t \leq \underline{a}$ or $t \geq \overline{a}$ and $a^{-1}(t) = w$, where $a(w) = t$ for $\underline{a} < t < \overline{a}$. Note that such a number w exists, since $a(\theta)$ is continuous. Similarly $b^{-1}(t)$ is defined. Since $a(\theta) < b(\theta)$, we have $b^{-1}(a(\theta)) < \theta$ and $\theta < a^{-1}(b(\theta))$. Thus we have $b^{-1}(\underline{a}) < b^{-1}(a(\theta)) < \theta < a^{-1}(b(\theta)) < a^{-1}(\overline{b})$. Hence $(b^{-1}(\underline{a}), a^{-1}(\overline{b}))$ is a trivial range for θ. In the following we assume that $g(x) > 0$ for $\underline{a} < x < \overline{b}$.

For $\underline{a} < x < \overline{b}$, define $G(x) = \int_0^x g(z)\, dz$ which is independent of θ. Let $Y = G(X)$, one to one transformation, then the p.d.f of Y is given by

$$f(y,\theta) = \begin{cases} 1/H(\theta), & A(\theta) < y < B(\theta);\ \theta \in \Theta \subset R \\ 0, & \text{otherwise,} \end{cases} \qquad (2.2)$$

where $A(\theta) = G(a(\theta))$, $B(\theta) = G(b(\theta))$ and $H(\theta) = B(\theta) - A(\theta)$. On the similar line as above, we can define $\underline{A}$, $\overline{A}$, $\underline{B}$, $\overline{B}$ and inverse functions $A^{-1}(t)$ as well as $B^{-1}(t)$ for every $t \in (\underline{A}, \overline{B})$. We note that G(x) is continuous, strictly increasing (as $g(x) > 0$). Hence $A(\theta)$, $B(\theta)$ are also continuous, strictly increasing and $H(\theta)$ is also positive. Note that the density of $Y = G(X)$ is of the form (2.1) with $g(.) = 1$. Hence if $g(x) \neq 1$ then by considering the p.d.f of G(X), the g(.) reduces to 1. Since $Y = G(X)$ is a one to one transformation, there is no loss of information.

Let $Y_1, Y_2, \ldots, Y_n$ be independent and identically distributed (i.i.d) random variables with p.d.f given by (2.2). Let $Y_{(1\,n)} = \min(Y_1, Y_2, \ldots, Y_n)$ and $Y_{(n\,n)} = \max(Y_1, Y_2, \ldots, Y_n)$. Since $A(\theta) < Y_{(1\,n)} < Y_{(n\,n)} < B(\theta)$, we have $B^{-1}(Y_{(n\,n)}) < \theta < A^{-1}(Y_{(1\,n)})$. Further as the functions $A(\theta)$, $B(\theta)$ are strictly increasing and continuous, $B^{-1}(Y_{(n\,n)})$ increases to θ a.s and $A^{-1}(Y_{(1\,n)})$ decreases to θ a.s. Thus ($B^{-1}(Y_{(n\,n)})$, $A^{-1}(Y_{(1\,n)})$) is a random interval that a.s 'contains θ and the length decreases to zero'.

Since the functions $A(\theta)$, $B(\theta)$ are continuous strictly increasing and $A(\theta) < B(\theta)$, we get $\underline{A} \leq \underline{B}$ and $\bar{A} \leq \bar{B}$. Further the observations fall in the interval ($\underline{A}$, $\bar{B}$) whatsoever $\theta \in (\underline{\theta}, \bar{\theta})$. If $\bar{A} \leq \underline{B}$ and all observations fall in the interval [$\bar{A}$, $\underline{B}$] then we get $B^{-1}(Y_{(n\,n)}) = \underline{\theta}$ and $A^{-1}(Y_{(1\,n)}) = \bar{\theta}$, otherwise $B^{-1}(Y_{(n\,n)}) > \underline{\theta}$ and $A^{-1}(Y_{(1\,n)}) < \bar{\theta}$. Hence to have a fixed width CI for θ, both $B^{-1}(Y_{(n\,n)})$ and $A^{-1}(Y_{(1\,n)})$ must be finite which implies both $\underline{\theta}$ and $\bar{\theta}$ must be finite. This can be described as below.

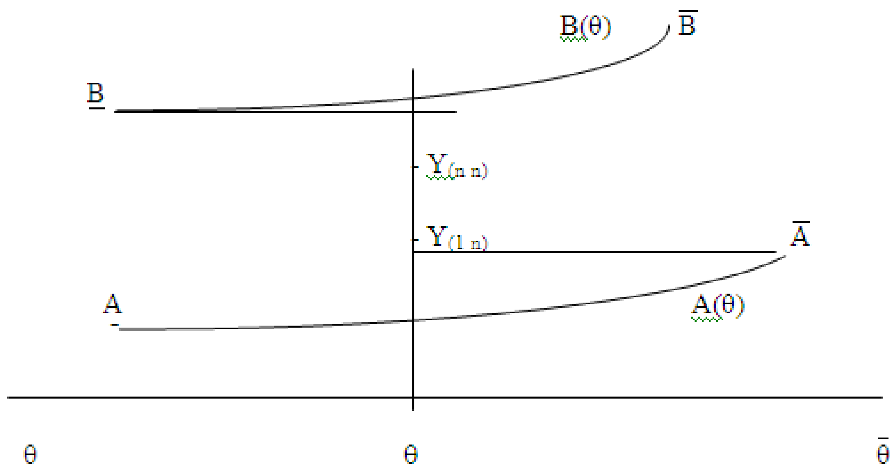

Figure 2.1: Nature of observations.

In the following for simplicity we assume that $\underline{\theta}$ and $\bar{\theta}$ are finite. If $A^{-1}(Y_{(1\,n)}) < \underline{\theta} + d$ or $B^{-1}(Y_{(n\,n)}) > \bar{\theta} + d$ then ($\underline{\theta}$, $A^{-1}(Y_{(1\,n)})$) or ($B^{-1}(Y_{(n\,n)})$, $\bar{\theta}$) constitute a 100% CI for θ of width at most d. Hence in the following we consider the case $A^{-1}(Y_{(1\,n)}) \geq \underline{\theta} + d$ and $B^{-1}(Y_{(n\,n)}) \leq \bar{\theta} + d$.

Let ($B^{-1}(Y_{(n\,n)})$, $B^{-1}(Y_{(n\,n)}) + d$) be a CI for θ of width d. The coverage probability of this interval is

$$P(B^{-1}(Y_{(n\,n)}) < \theta \leq B^{-1}(Y_{(n\,n)}) + d)$$
$$= P(Y_{(n\,n)} < B(\theta),\ \theta - d \leq B^{-1}(Y_{(n\,n)}))$$
$= P(\theta - d \leq B^{-1}(Y_{(n\,n)}))$ (since $Y_{(n\,n)} < B(\theta)$ almost surely(a.s)).
$= P(\max\{\theta - d, \underline{\theta}\} \leq B^{-1}(Y_{(n\,n)}))$ (since $\underline{\theta} \leq B^{-1}(Y_{(n\,n)})$ a.s) .
$$= \begin{cases} P(B^{-1}(Y_{(n\,n)}) \geq \theta - d) & \text{for } \theta - d \geq \underline{\theta} \\ P(B^{-1}(Y_{(n\,n)}) \geq \underline{\theta}) & \text{for } \theta - d < \underline{\theta} \end{cases}$$
$$= \begin{cases} P(Y_{(n\,n)} \geq B(\theta - d)) & \text{for } \theta - d \geq \underline{\theta} \\ 1 & \text{for } \theta - d < \underline{\theta} \end{cases} \qquad (2.3)$$

Thus if $\theta < \underline{\theta}$+d then the coverage probability of ($B^{-1}(Y_{(n\,n)})$, $B^{-1}(Y_{(n\,n)})$ + d) is 1. Now if $\theta \geq \underline{\theta}$+d the coverage probability of ($B^{-1}(Y_{(n\,n)})$, $B^{-1}(Y_{(n\,n)})$ + d) is

$$P(Y_{(n\,n)} \geq B(\theta-d)) = P(U_{(n\,n)} \geq [B(\theta-d)-A(\theta)]/H(\theta))$$
$$= \begin{cases} 1 & \text{if } B(\theta - d) \leq A(\theta), \\ 1 - \left[\dfrac{B(\theta - d) - A(\theta)}{H(\theta)}\right]^n & \text{if } B(\theta - d) > A(\theta). \end{cases} \qquad (2.4)$$

where $U_{(n,\,n)}$ is the largest of n i.i.d observations from U(0, 1). Thus from (2.3) and (2.4), for each n = 1, 2, …, the coverage probability of ($B^{-1}(Y_{(n,\,n)})$, $B^{-1}(Y_{(n,\,n)})$ + d) is 1 if $\theta < \underline{\theta}$+d or $B(\theta-d) \leq A(\theta)$ and it is strictly increasing in n to 1 for $B(\theta-d) > A(\theta)$ and $\theta \geq \underline{\theta}$ + d.

When $B(\theta-d) > A(\theta)$, for the coverage probability P($B^{-1}(Y_{(n\,n)}) < \theta < B^{-1}(Y_{(n\,n)})$ + d) to be at least 1-α , we must have,

$$1 - [(B(\theta - d)- A(\theta))/H(\theta)]^n \geq 1-\alpha,$$

which implies

$$n > \frac{\log(\alpha)}{\log\left[\dfrac{B(\theta-d)-A(\theta)}{H(\theta)}\right]} . = n(\theta) \qquad (2.5)$$

Thus for the coverage probability $P(B^{-1}(Y_{(n\,n)}) < \theta < B^{-1}(Y_{(n\,n)})$ + d) to be at least 1-α, the least fixed sample size n^* (as a function of θ) is

$$n^* := \begin{cases} \left[\dfrac{\log(\alpha)}{\log\left(\dfrac{B(\theta-d)-A(\theta)}{H(\theta)}\right)}\right] + 1 & \text{if } B(\theta-d) > A(\theta) \text{ and } \theta \geq \underline{\theta} + d \\ 1 & \text{if } B(\theta-d) \leq A(\theta) \text{ or } \theta < \underline{\theta} + d \end{cases}$$

where [x] indicates the least integer greater than or equal to x.

3. A PURELY SEQUENTIAL PROCEDURE

The advantages of sequential procedure are well known. Here by collecting observations one by one and by considering the accumulated information after each one, it could be possible to attain the desired precision with lesser number of observations. Also for any k, we have $B^{-1}(Y_{(k\ k)}) \le \theta \le A^{-1}(Y_{(1\ k)})$ a.s and hence in the following we propose a purely sequential procedure depending on values of $\underline{\theta}$, $\bar{\theta}$, $\bar{A}$ and $\underline{B}$.

3.1 Both $\underline{\theta}$ and $\bar{\theta}$ are finite, whatever may be $\bar{A}$ and $\underline{B}$

Let $Y_1, Y_2, \ldots$ be i.i.d random observations from (2.2). Stop after N observations, where N is the least integer n (≥ 1) such that

(i) $A^{-1}(Y_{(1\ n)}) - B^{-1}(Y_{(n\ n)}) \le d$,

OR

(ii) $$n \ge \sup_{\substack{B^{-1}(Y_{(n\ n)}) \le \theta \le A^{-1}(Y_{(1\ n)}) \\ B(\theta - d) > A(\theta)}} \left\{ \frac{\log(\alpha)}{\log\left(\frac{B(\theta - d) - A(\theta)}{H(\theta)}\right)} \right\} = n(\tilde{\theta}_n), \text{ say.} \qquad (3.1)$$

Since $B^{-1}(Y_{(n\ n)})$ and $A^{-1}(Y_{(1\ n)})$ are finite, so supremum is finite and is attained at some point $\tilde{\theta}_n$, say. Then the above stopping rule can be described as, N is the least integer n (≥ 1) such that

$$\left\{ A^{-1}(Y_{(1\ n)}) - B^{-1}(Y_{(n\ n)}) \le d \quad \text{OR} \quad n \ge \frac{\log(\alpha)}{\log\left(\frac{B(\tilde{\theta}_n - d) - A(\tilde{\theta}_n)}{H(\tilde{\theta}_n)}\right)} \right\} \qquad (3.1^*)$$

and propose the CI

$$C.I._N = (B^{-1}(Y_{(N\ N)}),\ \min\{A^{-1}(Y_{(1\ N)}), B^{-1}(Y_{(N\ N)}) + d\}). \qquad (3.2)$$

In the following we shall show that the rule N is closed and the coverage probability of $C.I._N$ is at least $1 - \alpha$.

Theorem 3.1.1: The stopping rule N is closed.
Proof: Let N_1 (≥ 1) be a stopping random variable corresponding to stopping rule $A^{-1}(Y_{(1\ n)}) - B^{-1}(Y_{(n\ n)}) \le d$. Then $P(N>k) \le P(N_1>k)$. As $B^{-1}(Y_{(k\ k)})$ increases to θ and $A^{-1}(Y_{(1\ k)})$ decreases to θ a.s, we have $A^{-1}(Y_{(1\ k)}) - B^{-1}(Y_{(k\ k)})$ decreases to 0 a.s as k tends to ∞. Hence $P(N_1 > k) = P[A^{-1}(Y_{(1\ k)}) - B^{-1}(Y_{(k\ k)}) > d]$ tends to 0 as k tends to ∞. Thus the stopping rule N_1 is closed. Hence stopping random variable N is closed. □
Theorem 3.1.2: The coverage probability of $C.I._N$, defined in (3.2), is at least $1 - \alpha$.

Proof: Let θ_0 be the true value of the parameter. Then for each $n \geq 1$, it is evident that $B^{-1}(Y_{(n\,n)}) \leq \theta_0 \leq A^{-1}(Y_{(1\,n)})$.

For $B(\theta_0-d) \leq A(\theta_0)$, there exist Y_1 such that $B^{-1}(Y_1) \leq \theta_0 \leq A^{-1}(Y_1)$ that is N =1 a.s and C.I.$_N$ has coverage probability 1.

If $B(\theta_0-d) > A(\theta_0)$ and $A^{-1}(Y_{(1\,N)}) - B^{-1}(Y_{(N\,N)}) \leq d$ then C.I.$_N$ = $(B^{-1}(Y_{(N\,N)}), \min\{A^{-1}(Y_{(1\,N)}), B^{-1}(Y_{(N\,N)}) + d\}) = (B^{-1}(Y_{(N\,N)}), A^{-1}(Y_{(1\,N)}))$ is a 100% CI for θ.

If $B(\theta_0-d) > A(\theta_0)$ and $A^{-1}(Y_{(1\,N)}) - B^{-1}(Y_{(N\,N)}) > d$ then by definition of N sampling is not stopped by condition (i), but by (ii) of (3.1), we have $N \geq n(\tilde{\theta}_n) > n(\theta_0)$, where $n(\theta_0)$ is as defined in (2.5). Moreover for each θ, $P_\theta\{B^{-1}(Y_{(n\,n)}) < \theta < B^{-1}(Y_{(n\,n)}) + d\} = 1 - [(B(\theta - d)-A(\theta))/H(\theta)]^n$ is increasing in n, we have,

$$P_{\theta_0}[\theta_0 \in C.I._N] \geq P_{\theta_0}[\theta_0 \in C.I._{n(\tilde{\theta}_n)}] \geq P_{\theta_0}[\theta_0 \in C.I._{n(\theta_0)}] \geq 1-\alpha.$$

In the above the last inequality follows from the definition of $n(\theta_0)$ defined in (2.5). Hence C.I.$_N$ has coverage probability1- α. □

3.2: One of $\underline{\theta}$ and $\bar{\theta}$ is not finite and $\bar{A} \leq \underline{B}$

If any one of the $\underline{\theta}$ and $\bar{\theta}$ is not finite and $\bar{A} \leq \underline{B}$ then for the interval $(B^{-1}(Y_{(n\,n)}), A^{-1}(Y_{(1\,n)}))$ to have finite length, one need to continue sampling

"until $Y_{(1\,n)} < \bar{A}$ if $\bar{\theta} = \infty$, or

until $Y_{(n\,n)} > \underline{B}$ if $\underline{\theta} = -\infty$ as the case(s) may be." (3.3)

Let N_2 be the stopping random variable corresponding to (3.3). Now stop if condition (3.1*) holds, otherwise continue sampling by taking an additional observation until (3.1*) holds. Let N be the total number of observations. Then in this case, the stopping random variable will be $\max(N_2, N) = N_3$ (say).

As n tends to ∞, $Y_{(n\,n)}$ tends to $B(\theta) > \underline{B}$ and $Y_{(1\,n)}$ tends to $A(\theta) < \bar{A}$. So condition (3.3) holds a.s. That is N_2 is closed. Also by theorem 3.1, N is shown to be closed. Hence N_3 is also closed.

Since $N_3 \geq N$ and by theorem 3.2, C.I$_N$ has coverage probability at least $(1-\alpha)$, C.I$_{N3}$ also has coverage probability at least $(1-\alpha)$.

3.3: One of $\underline{\theta}$ and $\bar{\theta}$ is not finite and $\bar{A} > \underline{B}$

In this case $B^{-1}(Y_{(n\,n)}) > \underline{\theta}$, $A^{-1}(Y_{(1\,n)}) < \bar{\theta}$ and the procedure described in Subsection 3.1 holds.

Remark 3.1: One may take ($A^{-1}(Y_{(n\ n)})$ - d, $A^{-1}(Y_{(n\ n)})$) as a CI for θ of width d. Then by appropriate modifications, the (1-α) level CI for θ will be (max{$B^{-1}(Y_{(N\ N)})$, $A^{-1}(Y_{(1\ N)})$ - d}, $A^{-1}(Y_{(1\ N)})$).

Remark 3.2: For the results of Section 2 and 3 to hold good, some of the assumptions in the model (2.1) can be relaxed to some extent, as indicated in the following.

(i) The condition of continuity can be relaxed. In this case define inverse function $a^{-1}(t)$ as $\underline{\theta}$, inf{θ: a(θ) ≥ t} or $\bar{\theta}$ according as $t < \underline{a}$, $\underline{a} \le t \le \bar{a}$ or $t > \bar{a}$ and define $b^{-1}(t)$ as $\underline{\theta}$, sup{θ: b(θ) ≤ t} or $\bar{\theta}$ according as $t < \underline{b}$, $\underline{b} \le t \le \bar{b}$ or $t > \bar{b}$. These inverse functions are defined so as to minimize $a^{-1}(t_1) - b^{-1}(t_2)$, the length of the interval ($b^{-1}(t_2)$, $a^{-1}(t_1)$), for any t_1, t_2 such that $\underline{a} \le t_1 < t_2 \le \bar{b}$.

(ii) If a(θ) and b(θ) are not strictly increasing but non decreasing functions and at each θ, at least one of them is strictly increasing (otherwise identification problem would have arise). In this case as a technical requirement it is enough to assume that for given fixed width d (> 0), there exist δ (> 0) such that $a^{-1}(a(\theta) + \delta) - b^{-1}(b(\theta) - \delta) < d$ for all θ.

For example in case of U(0, θ); θ > 0 model, $a^{-1}(t) = 0$ or ∞ according as $t \le 0$ or $t > 0$ and $b^{-1}(t) = 0$ or t according as $t < 0$ or $t \ge 0$. Thus the 100% CI will be ($X_{(n\ n)}$, ∞) for all n ≥ 1 and stopping rule (3.2*) can not be used. In such of the cases, the above assumption does not hold in the sense that the requirement is necessary.

4. ILLUSTRATION AND NUMERICAL EVALUATION

For an illustration of the proposed purely sequential procedure, we consider U(θ, mθ) model with m (>1) known and θ > 0. For U(θ, mθ) distribution, we have g(.) = 1, $\underline{\theta}$ = 0, $\bar{\theta}$ = ∞, A(θ) = θ and B(θ) = mθ. Here $A^{-1}(Y_{(1n)}) = Y_{(1\ n)}$, $B^{-1}(Y_{(n\ n)}) = Y_{(n\ n)}/m$ and H(θ) = (m-1)θ, which is positive. Also we have $\underline{A} = \underline{B} = 0$ and $\bar{A} = \bar{B} = \infty$. A purely sequential procedure consists of taking observations one by one and stop after N observations, where N **is** the least integer n (≥1) such that

(i) $Y_{(1\ n)} - Y_{(n\ n)}/m \le d$,

OR

$$\text{(ii) } n \ge \sup_{\substack{Y_{(n\ n)}/m < \theta < Y_{(1\ n)} \\ \theta > md/(m-1)}} \left\{ \frac{\log(\alpha)}{\log\left(1 - \frac{md}{(m-1)\theta}\right)} \right\}$$

For θ > md/(m-1), $\dfrac{\log(\alpha)}{\log\left(1 - \frac{md}{(m-1)\theta}\right)}$ is increasing function of θ and N is the least integer n (≥1) such that

$$\left\{ Y_{(1\,n)} - Y_{(n\,n)}/m \le d \quad \text{OR} \quad n \ge \left\{ \frac{\log(\alpha)}{\log\left(1 - \frac{md}{(m-1)(Y_{(1\,n)} \vee md/(m-1))}\right)} \right\} \right\}.$$

where ($Y_{(1\,n)}$ v md/(m-1)) means max{$Y_{(1\,n)}$, md/(m-1)}. The stopping rule in this case is max(N_2, N) = N_3 (say) and the (1-α) level CI for θ be ($Y_{(N\,N)}$/m, min{$Y_{(1\,N3)}$, $Y_{(N3\,\,N3)}$/m + d}).

If θ were known then in this case the fixed sample size to attain the level (1 - α) is

$$n^* := \begin{cases} \left[\frac{\log(\alpha)}{\log\left(1 - \frac{md}{(m-1)\theta}\right)} \right] + 1 & \text{if } \theta > md/(m-1) \text{ and } \theta \ge d. \\ 1 & \text{if } \theta \le md/(m-1) \text{ or } \theta < d. \end{cases}$$

where [x] denotes the least integer greater than or equal to x.

To verify the attainability of the coverage probability, in the following we carry out the simulation study for U(θ, mθ) model based on 10,000 iterations. Since all desired expression depend on θ and d through d/θ, in the following we fix d =1. The fixed sample size n^*, the simulated ASN and coverage probabilities are tabulated for different values of m, α and θ.

Table 4.1. The entries are respectively n^*, simulated 'ASN' and 'coverage probability' for purely sequential procedure when m = 2.

α\θ	1	2	10	50	100	300
0.01	1 1 1	1 2.78 1	22 13.30 0.9908	113 66.12 0.9937	228 120.29 0.9922	689 390.43 0.9901
0.05	1 1 1	1 2.45 1	14 11.27 0.9635	74 54.88 0.9508	149 108.13 0.9545	448 324.40 0.9508
01	1 1 1	1 2.17 1	11 9.67 0.9253	57 46.81 0.9053	114 92.56 0.9028	345 275.95 0.9009

Table 4.2. The entries are respectively simulated n^*, 'ASN' and 'coverage probability' for purely sequential procedure when m = 3

α\θ	1	2	10	50	100	300
0.01	1 2.28 1	3 4.61 0.9972	28 20.90 0.9925	151 101.72 0.9917	305 202.72 0.9914	919 601.64 0.9909
0.05	1 2.30 1	2 3.93 0.9855	18 16.97 0.9650	98 80.03 0.9538	198 158.59 0.9515	598 473.06 0.9508
0.1	1 1.82 1	2 3.47 0.9664	14 14.01 0.9263	76 66.70 0.9123	152 132.18 0.9099	459 392.75 0.9017

Remark 4.1 From Tables 4.1 - 4.2, it is clear that purely sequential procedure attains the desired level.
Remark 4.2 As expected, the ASN increases when θ and /or m increase.

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