Research Paper

Solving Quadratic Riccati Differential Equation using New Iterative Method

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Abstract. In this paper, we extend the application of New Iterative Method (NIM) proposed by Daftardar-Gejji and Jafari to solve Quadratic Riccati Differential Equation. We discussed numerical example to demonstrate the efficiency and the accuracy of the proposed Method.

Keywords: Quadratic Riccati Differential Equation, new iterative method

1. INTRODUCTION

In this paper, an analytical solution for the Riccati differential equation [7] will be discussed using of New Iterative Method (NIM)

$$\frac{dy}{dt} = Q(t)y + R(t)y^2 + P(t) \quad , \quad y(0) = G(t) \qquad (1)$$

where $Q(t)$,$R(t)$, $P(t)$,$G(t)$ are known scalar functions.

The Riccati differential equation is named after the Italian nobleman Count Jacopo Francesco Riccati (1676-1754). The book of Reid [8] contains the fundamental theories of Riccati equation, with applications to random processes, optimal control, and diffusion problems. Beside important engineering and science applications that today are known as the classical proved, such as stochastic realization theory, optimal control, robust stabilization, and network synthesis, the newer applications include such areas as financial mathematics [2, 6]. The solution can be find by classical numerical methods such as the forward Euler method and Runge-Kutta method. El-Tawil et al. [5] solved the nonlinear Riccati in an analytic form by Adomian decomposition method(ADM). Very recently, Tan and Abbasbandy [9] applied Homotopy Analysis Method (HAM) to solve a quadratic Riccati equation. Abbasbandy [1] solved one example of the quadratic Riccati differential equation (with constant coefficient) by He's variational iteration method by using Adomian's polynomials.

The paper is organized as follows : New iterative method described briefly in Section 2. Convergence results of NIM are stated in Section 3. Section 4 deals with illustrative examples and the conclusions are summarized in Section 5.

2. NEW ITERATIVE METHOD

The new iterative method (NIM) is described as bellow:
Consider the nonlinear equation [4]

$$u = f + N(u) \tag{2}$$

where f is a given function, and N is nonlinear operator from a Banach space $B \rightarrow B$. It is assumed that the NIM solution for the Eq. (3) has the form:

$$u = \sum_{i=0}^{\infty} u_i \tag{3}$$

The convergence of series (3) is proved in [3] and described in Section 4.
The nonlinear operator N in Eq. (3) is decomposed by Daftardar-Gejji and Jafari as bellow:

$$N(\sum_{i=0}^{\infty} u_i) = N(u_0) + \sum_{i=1}^{\infty} \left\{ N(\sum_{j=0}^{i} u_j) - N(\sum_{j=0}^{i-1} u_j) \right\} \tag{4}$$

From Eqs. (2) and (3), Eq. (4) is equivalent to

$$\sum_{i=0}^{\infty} u_i = f + N(u_0) + \sum_{i=1}^{\infty} \left\{ N(\sum_{j=0}^{i} u_j) - N(\sum_{j=0}^{i-1} u_j) \right\} \tag{5}$$

We define the following recurrence relation:

$$\begin{cases} G_0 = u_0 = f \\ G_1 = u_1 = N(u_0) \\ G_m = N(u_0 + u_1 + \dots\dots\dots + u_m) - N(u_0 + u_1 + \dots\dots + u_{m-1}) \end{cases} \tag{6}$$

Then

$$\sum_{i=1}^{m+1} u_i = N(\sum_{i=1}^{m} u_i) \tag{7}$$

and

$$u(x) = f + \sum_{i=1}^{i=\infty} u_i \tag{8}$$

The *m*-term approximate solution of equation (2) is given by $u = \sum_{i=0}^{m-1} u_i$

3. CONVERGENCE OF NIM

The following convergence results for NIM are described by Daftardar-Gejji and Bhalekar[3].

Theorem 1. If N is C^{∞} in a neighborhood of u_0 and $\left\| N^n(u_0) \right\| \le L$, for any n and for some real L > 0 and $\left\| u_i \right\| \le M < \frac{1}{e}$, i = 1, 2,…… , then the series $\sum_{n=0}^{\infty} G_n$ is absolutely convergent and Moreover, $\left\| G_n \right\| \le LM^n e^{n-1}(e-1)$, n=1,2,…….

Theorem 2. If N is C^{∞} and $\left\| N^n(u_0) \right\| \le M \le e^{-1}$, for all n, then $\sum_{n=0}^{\infty} G_n$ is absolutely convergent.

4. ILLUSTRATIVE EXAMPLES

Example 4.1. Consider the following equation [5]

$$\frac{dy}{dt} = -y^2(t) + 1$$

(9)

subject to the initial condition

$$y(0) = 0$$

(10)

Applying NIM, We get

$$y(t) = t - \int_0^t y^2(t)dt = f + N(y)$$

(11)

$$y_0 = t$$

(12)

$$y_1 = -\frac{t^3}{3}$$

(13)

$$y_2 = \frac{2}{15}t^5 - \frac{1}{63}t^7$$

(14)

The solution obtained using NIM approximated to 3 components:

$$y(t) \approx t - \frac{1}{3}t^3 + \frac{2}{15}t^5 - \frac{1}{63}t^7$$

Example 4.2. Consider the following quadratic Riccati differential equation [5]

$$\frac{dy}{dt} = 2y(t) - y^2(t) + 1$$

(15)

subject to the initial condition

$$y(0) = 0$$

(16)

Using NIM we get

$$y(t) \approx t - t^2 - \frac{1}{3}t^3 - \frac{1}{3}t^4 - \frac{3}{5}t^5 + \ldots\ldots\ldots\ldots\ldots$$

(17)

5. CONCLUSIONS

We have successfully applied new iterative method (NIM) to obtain approximate solutions of Quadratic Riccati Differential Equation. The result shows that NIM is very simple but powerful technique used to solve such type of equations. The solutions obtained by NIM are in good agreement with solutions Obtained by ADM [5]. Moreover, the accuracy is high with little computed terms of the solution

6. REFERENCES

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