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FUZZY IDEALS IN Γ-NEAR-SUBTRACTION SEMIGROUPS

Dr. Alandkar Sanjay Jaykumar
Head and Associate Professor, Department of Mathematics and Statistics. Walchand College of Arts and Science, Solapur [M.S.]

ABSTRACT

In this paper,we introduce a concept of fuzzy ideals in Γ-near-subtraction semigroups and discuss some of its properties.

KEY WORDS: Γ- near subtraction semigroup, Ideals of Γ- near subtraction semigroup, fuzzy level set, fuzzy ideal, fuzzy homomorphism.

I. INTRODUCTION

B. M. Schein [10] introduced the system (N, ◦ , -.), where N is a set of functions closed under the composition "◦" of functions (and hence (N, ◦) is a function semigroup) and the set theoretic subtraction '-' (and so (N, -) is a subtraction algebra in the sense of [1]). B. Zelinks[13] studied subtraction semigroup. In [3, 4], Y. B. Jun and H. S. Kim discussed the ideals of a set and its related results. Pilz[8] has developed near-ring theory. Dheena at el. [2] defined the near subtraction semigroups . A graded membership of set is newly introduced by L. A. Zadeh [11].

Y. S. Pawar at el [7] discussed ideals and its properties , H. V. Kumbhojkar at el [5] studied correspondences of fuzzy ideals. K.J. Lee and C.H. Park[6] introduced the notion of a fuzzy ideal in subtraction algebras, and give some conditions for a fuzzy set to be a fuzzy ideal in subtraction algebras. Satyanarayan [9] defined the concept of Γ- near ring and their ideals. D. R. Prince Williams[11] studied Fuzzy ideals in near-subtraction semigroup. S. J. Alandkar[1] generalize the concept of near subtraction semigroups and introduce the concept of Γ-near subtraction semigroup[1] and studied its properties. In this paper, we fuzzificate the concept of Γ-near subtraction semigroup[1] and study its properties.

II. PRELIMINARIES

We recalled the following definitions and its properties :

Definition 2.1. Let A be a nonempty set and subtraction '-' is a single binary operation. Then an algebra is said to be a subtraction algebra (A, −) if it satisfies the following axioms: for any a, b, c ∈ A,

(i) a − (b − a) = a;
(ii) a − (a − b) = b − (b − a);
(iii) (a − b) − c = (a − c) − b.

In (iii) omition of parentheses in expressions of the form (x − y) – z is allowed.

In a subtraction algebra, the following properties are satisfied:

P1. a −0 = a and 0 − a = 0.
P2. a − (a − b) =0.
P3. (a − b) − b = (a − b).
P4. (a−b)− (b − a)= a – b where a–a = 0 where element a does not depend on the choice of a∈ A.

Definition 2.2. A non empty subset S of a subtraction algebra N is said to be a subalgebra of N, if a − b ∈ S whenever a, b ∈ S.

Definition 2.3. A subtraction semigroup is an algebra (A, ·, −) with two binary operations '-'and '.'that satisfies the following properties: for any a, b, c ∈ A,

1. (A, .) is a semigroup,
2. (A, -) is a subtraction algebra,
3. a(b − c) = ab -ac and (a − b)c = ac - bc.

A subtraction semigroup is said to be multiplicatively abelian if multiplication is commutative.

Definition 2.4. A non-empty set N together with the binary operations "–" and "." is said to be a near-subtraction semigroup if it satisfies the following:

1. (N, -) is a subtraction algebra.
2. (N, .) is a semigroup.
3. (a − b)c = ac − bc,for all a, b,c ∈ N.

It is clear that 0a = 0,for all a ∈ N. Similarly we can define a near-subtraction semigroup (left).

We always take a near-subtraction semigroup means it is a near-subtraction semigroup(right) only.

Definition 2.5. Let (N, -) be a near-subtraction semigroup and Γ ={ α, β, ..} be a nonempty set of operators. Then N is said to be a Γ-near subtraction semigroup, if there exists a mapping N × Γ × N → N (the image of (a, α, b) is denoted by aαb), satisfying the following conditions:

1. (N, α) is a semigroup, α ∈ Γ
2. (N, -) is a subtraction algebra,
3. (a − b)αc = aαc – bαc (right distributive law),
4. (aαb)βc = aα(bβc) for all a, b, c ∈ M and α, β ∈ Γ.

Definition 2.5. Let (N, -) be a near-subtraction semigroup and Γ ={ α, β, ..} be a nonempty set of operators. Then N is said to be a Γ-near subtraction semigroup, if there exists a mapping N × Γ × N → N (the image of (a, α, b) is denoted by aαb), satisfying the following conditions:

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4. (aαb)βc = aα(bβc) for all a, b, c ∈ M and α, β ∈ Γ.

In practice we called simply 'Γ-near subtraction semigroup' instead of 'right Γ-near subtraction semigroup' . Similarly we can define a Γ-near subtraction semigroup (left). It is clear that 0αa = 0,for all a ∈ N and α ∈ Γ.

Example 2.6. Let N = {0, 1, 2, 3, 4, 5} in which '-' and α ∈ Γ are defined by

-	0	1	2	3	4	5
0	0	0	0	0	0	0
1	1	0	3	4	3	1
2	2	5	0	2	5	4
3	3	0	3	0	3	3
4	4	0	0	4	0	4
5	5	5	0	5	5	0

A	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	4	3	4	0
2	0	4	2	0	4	5
3	0	3	0	3	0	0
4	0	4	4	0	4	5
5	0	0	5	0	0	5

β	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	4	3	4	0
2	0	4	2	0	4	5
3	0	3	0	3	0	0
4	0	4	4	0	4	5
5	0	0	5	0	0	5

It is easily verified that N is a Γ-near subtraction semigroup.

Notations : In this paper, N denotes, unless otherwise stated, a Γ-near-subtraction semigroup. Here we consider the following notations

1. N∗ denotes the set of all non-zero elements of N. That is N∗ = N − {0}.
2. N0 = {a ∈ N/aα0 = 0} is called the zero symmetric part of N. N is called zero symmetric if N = N0.
3. Nd = {n ∈ X/nα(a − a′) = nαa − nαa′ for all a, a′ ∈ N} - the set of all distributive elements of N. N is called distributive if N = Nd.
4. The centre of N is defined as C(N) = {a ∈ N/aαk = kαa for all k ∈ N }.
5. T denotes the set of all idempotent elements of T (t ∈ T if and only if $t^2 = t$).
6. V denotes the set of all nilpotent elements of N (a ∈ V if and only if $a^k = 0$ for some positive integer k).

In a right Γ- near subtraction semigroup N, 0αa = 0 for all a ∈ N. But aα0 need not be equal to 0, for a ∈ N. Accordingly, we define the following:

Definition 2.7. (i) The set {a ∈ N/aα0 = 0} is called the zero-symmetric part of N and is denoted by N0.
(ii) A right Γ- near subtraction semigroup N is said to be zero symmetric if N = N0.
Example 2.6. verifies that (N,−, α) for α ∈ Γ is a zero symmetric right Γ- near subtraction semigroup i.e., N = N0.
Now we introduce the ideals of Γ-near subtraction semigroup.

Definition 2.8. Let (N,−, α) for α ∈ Γ be a Γ-near subtraction semigroup. A non empty subset I of N is called
(i) a left ideal if I is a subalgebra of (N,−) and aαi − aα(a′ − i) ∈ I for all a, a′ ∈ N and i ∈ I
(ii) a right ideal if I is a subalgebra of (N,−) and IΓN ⊆ I
(iii) an ideal if I is both a left ideal and a right ideal.
Here in Example 2.4, I = {0, 1, 2, 3, 4} is an ideal of N.

Remark 2.9. (i) Suppose N is a zero symmetric Γ-near subtraction semigroup and I is a left ideal of N such that a –b for every a, b ∈ N Then the following are equivalent:
(i) NΓI ⊆ I
(ii) aαi−aα(a′−i) ∈ I for all a, a′ ∈ N, α ∈ Γ and i ∈ I.

Definition 2.10. A mapping μ : N → [0, 1] is called **fuzzy set** of N.

Definition 2.11. The **complement** of a fuzzy set μ, denoted by $\bar{\mu}$ is the fuzzy set in N given by $\bar{\mu}(a) = 1 - \mu(a)$ for all a∈ N.

Definition 2.12. The level set of a fuzzy set μ of N is defined as $\mu_t^{\geq}(a) = \{a \in N \mid \mu(a) \geq t\}$, for all $0 \leq t \leq 1$.

Throughout this paper N denote a Γ-near-subtraction semigroup unless otherwise specified.

III. FUZZY IDEALS OF A Γ-NEAR-SUBTRACTION SEMIGROUP

Definition 3.1: A fuzzy set μ in N is called a fuzzy ideal of N if it satisfies the following conditions:
(GFI1) $\mu(a - b) \geq \min\{\mu(a), \mu(b)\}$ for all a, b ∈ N,
(GFI2) $\mu(a\alpha x - a\alpha(b - x)) \geq \mu(x)$ for all a, b, x ∈ N, α ∈ Γ and
(GFI3) $\mu(a\alpha b) \geq \mu(a)$, for all a, b∈ N, α ∈ Γ.
Note that μ is a fuzzy left ideal of N if it satisfies(GFI1)and(GFI2), and μ is a fuzzy right ideal of N if it satisfies (GFI1) and (GFI3).

Example 3.2. Using Example 2.4 , For α ∈ Γ , (N,−, α) is a Γ-near-subtraction semigroup and I = {0, 1, 2, 3, 4}. Let μ be a fuzzy set on N defined by μ(0) = 0.8, μ(a) = 0.5 for all a ∈ I and μ(5) = 0.3.Then it can be easily verified that μ is a fuzzy ideal of N.

Theorem 3.3: Let μ be a fuzzy left (resp. right) of N.Then the set $N\mu = \{a \in N \mid \mu(a) = \mu(0)\}$is a left(resp.right) ideal of N.

Proof: Suppose μ is a fuzzy left ideal of N and let a, b ∈ Nμ.Then $\mu(a - b) \geq \min\{\mu(a), \mu(b)\} = \mu(0)$.Thus a − b ∈ Nμ.
For every a, b ∈ N, α ∈ Γ and x ∈ Nμ,we have $\mu(a\alpha x - a\alpha(b - x)) \geq \mu(x) = \mu(0)$.Thus $a\alpha x - a\alpha(b - x) \in N\mu$. Hence, Nμ is a left ideal of N.
For every a, b ∈ N, α ∈ Γ and x ∈ ,we have $\mu(a\alpha b)) \geq \mu(a) = \mu(0)$.Thus aαb ∈ Nμ. Hence, Nμ is a right ideal of N.
Thus Nμ is an ideal of N.

Theorem 3.4: Let A be a non-empty subset of N and μ_A be a fuzzy set in N defined by

$\mu_A(a) = \begin{cases} s, & \text{if } a \in A \\ t, & otherwise \end{cases}$ for all a∈ N and s, t ∈ [0, 1] with s > t.

Then μ_A is a fuzzy ideal of N if and only if A is an ideal of N. Moreover $N_{\mu_A} = A$.

Proof: Suppose μ_A is a fuzzy ideal of N. Let a, b ∈ A.
Then $\mu(a - b) \geq \min\{\mu(a), \mu(b)\} = s$. Thus, a − b∈ A.
For every a, b ∈ N, α ∈ Γ and x ∈ A,we have $\mu(a\alpha x - a\alpha(b - x)) \geq \mu(x) = s$
Thus $a\alpha x - a\alpha(b - x) \in A$.
For all a, b ∈ A, α ∈ Γ.Then $\mu(a\alpha b) \geq \mu(a) = s$. Thus, aαb ∈ A.Hence , μ_A is an ideal of X.
Conversely, assume that A is an ideal of N. Let x, y ∈ N. If at least one of N and y does not belong to A,then $\mu_A(x - y) \geq t = \min\{\mu_A(x), \mu_A(y)\}$.
If a, b ∈ A then a − b ∈A ,we have $\mu_A(a - b) \geq s = \min\{\mu_A(a), \mu_A(b)\}$.
Let a, b, x ∈ N, α ∈ Γ and if x ∈ A such that aαx−aα(b−x) ∈ A, we have $\mu_A(a\alpha x - a\alpha(b - x)) \geq s = \mu_A(x)$.
If x ∉ A such that aαx − aα(b − x) ∉ A,we have $\mu_A(a\alpha x - a\alpha(b - x)) \geq t = \mu_A(x)$.
For all a, b ∈ A, α ∈ Γ then aαb ∈ A, we have $\mu_A(a\alpha b) \geq s = \mu(a)$.
Suppose a∉ A we have $\mu_A(a\alpha b) \geq t = \mu(a)$.
Hence μ_A is a fuzzy ideal of N. Also $N_{\mu_A} = \{x \in N \mid \mu_A(x) = \mu_A(0)\} = \{x \in N \mid \mu_A(x) = s\} = \{x \in N \mid x \in A\} = A$.

Corollary 3.5: Let χ_A be the characteristic function of a subset A ⊆ N.Then χ_A is a fuzzy left(resp. right) ideal if and only if A is a left(resp. right) ideal.

Theorem 3.6: Let μ be a fuzzy subset of N.Then μ is a fuzzy ideal of N if and only if each non-empty level subset $\mu_t^{\geq}$ of μ is an ideal of N.

Proof: Assume that μ is a fuzzy ideal of N and $\mu_t^{\geq}$ is a non-empty level subset of N.
(i) Since $\mu_t^{\geq}$ is a non-empty level subset of μ, there exists a, b∈ $\mu_t^{\geq}$, $\mu(a-b) \geq \min\{\mu(a), \mu(b)\} = t$. Thus $a-b \in \mu_t^{\geq}$.
(ii) Let a, b, x ∈ $\mu_t^{\geq}$, α ∈ Γ ,we have $\mu(a\alpha x - a\alpha(b-x)) \geq \mu(x) \geq t$.Thus $a\alpha x - a\alpha(b - x) \in \mu_t^{\geq}$.
(iii) Let a, b ∈ $\mu_t^{\geq}$, such that $\mu(a\alpha b) \geq \mu(a) \geq t$.Thus $a\alpha b \in \mu_t^{\geq}$. Hence, $\mu_t^{\geq}$ is an ideal of N.
Conversely,suppose that $\mu_t^{\geq}$ is an ideal of N.
(i)Let if possible, $\mu(a_0 - b_0) < \min\{\mu(a_0), \mu(b_0)\}$,for some $a_0, b_0 \in \mu_t^{\geq}$,then by taking

$t_0 = (1/2) \{\mu(a_0 - b_0) + \min\{\mu(a_0), \mu(b_0)\}\}$,

we have $\mu(a_0 - b_0) > t_0$,for $\mu(a_0) \geq t_0$, $\mu(b_0) \geq t_0$.Thus $a_0 - b_0 \notin \mu_t^{\geq}$,for some $a_0, b_0 \in \mu_t^{\geq}$.This is a contradiction, and so $\mu(a-b) \geq \min\{\mu(a), \mu(b)$,for all $a, b \in \mu_t^{\geq}(a)$.
(ii)Let if possible, for some $a_0 \in \mu_t^{\geq}$ $\mu(a\alpha x-(a\alpha(b-x)) < \mu(a_0)$, for all a, b ∈ N, α ∈ Γ and ,then by taking

$t_0 = (1/2)\{\mu(a\alpha x_0 - a\alpha(b - x_0)) + \mu(x_0)\}$,

we have $\mu(a\alpha x_0 - a\alpha(b - x_0)) > t_0$,for $\mu(a_0) \geq t_0$, $\mu(b_0) \geq t_0$.Thus $a\alpha x_0 - a\alpha(b-x_0) \notin \mu_t^{\geq}$,for some $x_0 \in \mu_t^{\geq}$ and for all a, b ∈ N. This is a contradiction, and so $\mu(a\alpha x - a\alpha(b - x)) \geq \mu(x)$, for all $x \in \mu_t^{\geq}$ and a, b ∈ N.
(iii) Let if possible, $\mu(a_0\alpha b_0) < \mu(a_0)$,for some $a_0, b_0 \in \mu_t^{\geq}$,then by taking

$t_0 = (1/2) \{\mu(a_0\alpha b_0) + \mu(a_0)\}$, we have $\mu(a_0\alpha b_0) > t_0$,for $\mu(a_0) \geq t_0$, $\mu(b_0) \geq t_0$.Thus $a_0\alpha b_0 \notin \mu_t^{\gtrless}$,for some a_0, $b_0 \in \mu_t^{\gtrless}$. This is a contradiction, and so $\mu(a\alpha b) \geq \mu(a)$,for all a, b $\in \mu_t^{\gtrless}$. Hence $\mu_t^{\gtrless}$ is a fuzzy ideal of N.

Definition 3.7: Let N be a Γ-near-subtraction semigroup and a_family of fuzzy sets {μ_i | i ∈ Z} in N. Then the intersection $\bigcap_{i=1}^{n} \mu_i$ of {μ_i | i ∈ Z} is defined by

$$\bigcap_{i=1}^{n} \mu_i(a) = \inf \{\mu_i(a) | a \in N, \}$$

Theorem 3.8: If {μ_i|i ∈ I} is a family of fuzzy ideal of N, then $\bigcap_{i=1}^{n} \mu_i(a)$ is a fuzzy ideal of N.

Proof: Let {μ_i|i ∈ I} be a family of fuzzy ideal of N.

(i)For all a, b ∈ N,we have

$(\bigcap_{i=1}^{n}) \mu_i(a - b) = \inf \{\mu_i(a - b) | i \in I\}$

$\geq \inf \{\min (\mu_i(a), \mu_i(b)) | i \in I\}$

$= \min \{\inf (\mu_i(a) | i \in I) , \inf (\mu_i(b) | i \in I)\}$

$= \min\{ \bigcap_{i=1}^{n} \mu_i(a) , \bigcap_{i=1}^{n} \mu_i(b) \}$

(ii) For all a, b, x ∈ X,we have

$(\bigcap_{i=1}^{n} \mu_i)(a\alpha x - a\alpha(b - x))$

$= \inf \{\mu_i(a\alpha x - a\alpha(b - x)) | i \in I\}$

$\geq \inf \{\mu_i(x) | i \in I\}$

$= \{\inf (\mu_i(x) | i \in I)\}$

$= (\bigcap_{i=1}^{n}) \mu_i(x)$.

(iii) For all a, b ∈ N, we have

$(\bigcap_{i=1}^{n}) \mu_i(a\alpha b)$

$= \inf \{\mu_i(a\alpha b) | i \in I\}$

$\geq \inf \{\min (\mu_i(a)) | i \in I\}$

$= (\bigcap_{i=1}^{n}) \mu_i(a)$

Hence $(\bigcap_{i=1}^{n}) \mu_i(a)$ is a fuzzy ideal of N.

Definition 3.9: Let f : N → N' be a mapping ,where N and N' are non-empty sets and μ is a fuzzy subset of N. The preimage of μ under f written μ^f, is a fuzzy subset of N defined by $\mu^f = \mu(f(a))$,for all a ∈ N.

Theorem 3.10: Let f : N→ N' be a homomorphism of Γ-near-subtraction semigroups. If μ is a fuzzy ideal of N_,then μ^f is a fuzzy ideal of N.

Proof: Suppose μ is a fuzzy ideal of N, then

(i) For all a, b ∈ N,we have

$\mu^f (a - b) = \mu(f(a - b))$

$= \mu(f(a) - f(b))$

$\geq \min\{\mu(f(a)), \mu(f(b))\}$

$= \min\{\mu^f (a), \mu^f(b)\}$.

(ii) For all a, b, x ∈ N,

we have $\mu^f (a\alpha x - a\alpha(b - x)) = \mu (f(a\alpha x - a\alpha(b - x)))$

$= \mu(f(a\alpha x) - f(a\alpha(b - x)))$

$= \mu(f(a)\alpha f(x) - f(a)\alpha(f(b) - f(x)))$

$\geq \mu(f(x))$

$= \mu^f (x)$.

(iii)For all a, b ∈ N,we have $\mu^f (a\alpha b) = \mu(f (a\alpha b))$

$= \mu(f(a)\alpha f(b))$

$\geq \mu (f(a))$

$= \mu^f(y)$.

Hence μ^f is a fuzzy ideal of N.

Theorem 3.11: Let f : N→ N' be a homomorphism of Γ-near-subtraction semigroup . If μ^f is a fuzzy ideal of N, then μ is fuzzy ideal of N'.

Proof: Suppose μ is a fuzzy ideal of X_,then

(i) Let a', b' ∈ N', there exists a, b ∈ N such that f(a) = a' and f(b) = b', we have

$\mu(a' - b') = \mu(f(a) - f (b)) = \mu(f(a - b)) = \mu^f(a - b) \geq \min \{ \mu^f (a), \mu^f (b)\} = \min\{\mu(f(a)) , \mu(f(b))\}= \min\{\mu(a'), \mu(b')\}$.

(ii)Let a', b', x' ∈ N',there exists a, b, x ∈ N such that f(a) = a', f(b) = b' and f(x) = x', we have $\mu (a'\alpha x' - b\alpha(a' - x')) = \mu(f(a)\alpha f(x) - f(b)\alpha(f(a) - f(x))) = \mu(f(a\alpha x) - f(b)\alpha f(a - x)) = \mu(f(a\alpha x) - f(b\alpha(a - x))) = \mu (f(a\alpha x - b\alpha(a-x))) = \mu^f (a\alpha x - b\alpha(a - x)) \geq \mu^f (x) = \mu(f(x)) = \mu(x')$.

(iii)Let a', b' ∈ N', there exists a, b ∈ N such that f(a) = a' and f(b) = b',

we have $\mu (a'\alpha b') = \mu(f(a)\alpha f(b)) = \mu(f(a\alpha b)) = \mu^f(a\alpha b) \geq \mu^f (a) = \mu(f(a)) = \mu(a')$.

Hence μ is a fuzzy ideal of N'.

Definition 3.12: Let f be a mapping defined on N. If ν is a fuzzy subset in f(N),then the fuzzy subset $\mu = \nu \circ f$ in N(i.e., the fuzzy subset defined by $\mu(a) = \nu(f(a))$ for all a ∈ N) is called the preimage of ν under f.

Proposition 3.13: An onto homomorphic preimage of a fuzzy ideal of N is a fuzzy ideal.

Proof: Straight forward.

Let μ be a fuzzy subset in N and f be a mapping defined on N.Then the fuzzy subset μf in f(X) defined by $\mu^f(y) = \sup_{a\in f^{-1}(b)} \mu(a)$ for all b ∈ f(N) is called the image of μ under f. A fuzzy subset μ in N is said to have an sup property if for every subset M ⊆ N, there exists n_0 ∈ M such that $\mu(n_0) = \sup_{n\in M} \mu(n)$.

Proposition 3.14: An onto homomorphic image of a fuzzy ideal with sup property is fuzzy ideal.

Proof: Let f : N → N' be an onto homomorphism of Γ-near subtraction semigroup and let μ be a fuzzy ideal of N with the sup property.

(i)Given a', b'∈ N', we let $a_0 \in f^{-1}(a')$ and $b_0 \in f^{-1}(b')$ be such that $\mu(a_0) = \sup_{n\in f^{-1}(a')} \mu(n)$ sup , $\mu(b_0) = \sup_{n\in f^{-1}(b')} \mu(n)$ sup respectively.Then , we have

$\mu^f(a' - b') = \sup_{m\in f^{-1}(a')} \mu(m) \geq \min\{\mu(a_0), \mu(b_0)\} = \min\{\sup_{n\in f^{-1}(a')} \mu(n), \sup_{n\in f^{-1}(b')} \mu(n)\}$

$= \min\{\mu^f(a'), \mu^f(b')\}$

(ii) Given a', b', x' ∈ N', we let $a_0 \in f^{-1}(a')$, $b_0 \in f^{-1}(b')$, $x_0 \in f^{-1}(x')$ be such that $\mu^f(a'\alpha x' - a'\alpha(b' - x')) = \sup_{m\in f^{-1}(a'\alpha x' - a'\alpha(b' - x')))} \mu(m)$

$\geq \mu(a_0)$

$= \sup_{m\in f^{-1}(a')} \mu(m)$

$= \mu^f(a')$.

(iii)Given a', b' ∈ N', we let $a_0 \in f^{-1}(a')$ and $b_0 \in f^{-1}(b')$ be such that

$\mu(a_0) = \sup_{m\in f^{-1}(a')} \mu(m)$, $\sup_{m\in f^{-1}(b')} \mu(m)$

respectively.Then , we have

$\mu^f(a'\alpha b') = \sup_{m\in f^{-1}(a'\alpha b')} \mu(m)$

$\geq \mu(a_0)$

$= \sup_{m\in f^{-1}(a')} \mu(m)$

$= \mu^f(a')$.

Hence, μ^f is a fuzzy ideal of N'.

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Dr. Alandkar Sanjay Jaykumar

Head and Associate Professor, Department of Mathematics and Statistics. Walchand College of Arts and Science, Solapur [M.S.]