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PSEUDO IDEALS OF A Γ -SEMI NEAR RING

S. J. Alandkar

Head and Associate Professor, Department of Mathematics & Statistics, Walchand College of Arts and Science, Solapur [M.S.]

Abstract: In this paper, the concept of pseudo ideal of a Γ-semi near ring is introduced and investigated some of its properties.

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§1. INTRODUCTION.

G. Pilz [4] studied Near rings and defined it as: if in a ring we ignore the commutativity of addition and one distributive law. The concept of Γ- ring was introduced by Nobusawa [2] and a generalization of the concept**,** Γ- near ring was introduced by Satyanarayana [8]. Further M. K. Rao [6] studied Γ-semir ring and then S. Pianskool at el [5] and N. K. Saha at el [7, 10] defined the generalization of Γ-semir ring and Γ- near ring as a Γ- semi-near rings and studied its properties. Various types of ideals in near rings and Γ- near ring are studied in articles [9, 11].

Berman G. and Silverman R. J.[1] have initiated a study of pseudo ideals of a near ring. A nonempty subset I of a near ring N is a left pseudo ideal of N if $(ni - n0) \in I$; a right pseudo ideal of N if $(in) \in I$ for all $i \in I$, $n \in N$ and a pseudo ideal of N if it is both a left pseudo ideal and a right pseudo ideal.

In this Paper, we introduce the concept of pseudo ideals of a Γ- semi near ring and study its properties. Throughout this paper M denotes a right Γ-seminear ring and we shall call it Γ-seminear ring only unless otherwise specified.

§2.1. Pseudo ideals of a Γ-seminear ring.

We begin with the following definition.

Definition 2.1.1. Seminear ring:-A nonempty set N together with two binary operations '+' and '·' satisfying the following conditions, is said to be a seminear ring.

i) (N, +) is a semigroup,
ii) (N, ·) is a semigroup,
iii) $(x+y)\cdot z = x\cdot z + y\cdot z$ for all $x, y, z \in N$.

Precisely speaking 'seminear ring' is a 'right seminear ring' here since every seminear ring satisfy one distributive law (left / right distributive law).

Every near ring is a seminear ring but every seminear ring need not be a near ring. For this we consider the following Example.

Example 1:-Let $N = \{\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}$ / a, b be nonnegative integers}, (N, +, .) is seminear ring under the matrix addition and matrix multiplication. Here N is a seminear ring which is not a near ring since (N, +) is a semigroup but not a group since additive inverse does not exist for all members of N.

Definition 2.1.2. Γ-near ring:-Let (M, +) be a group (need not be abelian) and Γ be a nonempty set. Then M= (M, +, Γ) is a Γ-near ring if there exists a mapping $M\times \Gamma \times M\rightarrow M$ (the image of $(x, \alpha, y)\rightarrow x\alpha y$) satisfying the following conditions :

i) (M, +, α) is a right near ring,
ii) $x\alpha (y\beta z) = (x\alpha y) \beta z$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

Precisely speaking 'Γ-near ring' is a 'Γ-near ring'. Every near ring is a special type of Γ-near ring for singleton set Γ whereas every Γ-near ring a near ring for each member of Γ. See the following example.

Example 2: Let $G = Z_8 = \{0, 1, 2, ..7\}$, the additive group of integers modulo 8 and X= {a, b}. Define m_i: X⟶G, $m_i(a) = 0$, $m_i(b) = i$, for $0\leq i \leq 7$. such that $M =\{m_0 ,m_1 ,..., m_7\}$ and let $\Gamma = \{ g_0 ,g_1\}$ where g_i :G⟶X define by

$$g_0 = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ a & a & a & a & a & a & a & a \end{pmatrix}, \quad g_1 = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ a & a & a & b & a & a & a & b \end{pmatrix}$$

For $m \in M$, $g \in \Gamma$, $x \in G$. Take $mgx = m(g(x))$.
Then (M, +, Γ) becomes Γ-near-ring.

Definition 2.1.3. Γ-seminear ring:-Let M be an additive semigroup and Γ be a nonempty set. Then a semigroup (M, α) is called a right Γ-seminear ring if there exists a mapping $M \times \Gamma\times M \rightarrow M$ (denoted by $(a, \alpha, b) \rightarrow a\alpha b$) satisfying the conditions:

i) $(a+b) \alpha c = a\alpha c+b\alpha c$,
ii) $a\alpha(b\beta c) = (a\alpha b)\beta c$

for all a, b, c ∈ M and α, β ∈ Γ. Precisely speaking 'Γ-seminear ring' to mean 'right Γ-seminear ring'.

Every Γ-near ring is a Γ-seminear ring but every Γ-seminear ring need not be a Γ-near ring. For this we consider the following Example.

Example 3: Let M = $\{\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}$ / a, b be nonnegative integers} = Γ, Then (M, +, Γ) is Γ- seminear ring under the matrix addition and matrix multiplication. Here N is a seminear ring which is not a near ring since (M, +) is a semigroup but not a group since additive inverse does not exist for all members of M. Define M × Γ× M → M (denoted by (a, α, b) → aαb) where aαb is matrix multiplication of a, α, b Then M is a Γ-seminear ring but not a Γ-near ring. Since (M, +) is a semigroup which is not a group.

Definition 2.1.4. Sub-Γ-seminear ring:- Let M be a Γ-seminear ring. A nonempty subset M′ of M is a sub-Γ-seminear ring of M if M′ is also a Γ-seminear ring with the same operations of M.

Definition 2.1.5. Ideal of a Γ-seminear ring:-A subset I of a Γ-seminear ring M is a left (resp. right) ideal of a Γ-seminear ring M if I is a subsemigroup of M and rαx ∈I(resp. xαr ∈I) for all x, y ∈ I and r ∈ M, α ∈ Γ.

If I is both left as well as right ideal then we say that I is an ideal of M.

Example 4: Consider the example of Γ-seminear ring (M, +, .) mentioned above. We have

I=$\{\begin{bmatrix} 2a & 2b \\ 0 & 0 \end{bmatrix}$ / a, b be nonnegative integers} is an ideal of M.

Definition 2.1.6 : A subset I of M is a left (resp. right) pseudo ideal of M if
(i) (I, +) is a normal subgroup of (M, +),
(ii) xαa-xα0 ∈ I (aαx∈ I) for all a∈ I, x, y∈ M and α ∈Γ.
(iii)

Example 5: Let (M, +) be an additive group such that M = {0, a, b, c} and Γ = {α, β} be a nonempty set of binary operations on M as shown in the following tables.

β	0	a	b	c
0	0	0	0	0
a	0	0	0	0
b	0	0	0	0
c	0	0	0	0

α	0	a	b	c
0	0	0	0	0
a	0	0	a	a
b	0	a	b	b
c	0	a	c	c

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Then (M, +, Γ) is a Γ-semi near ring and I = {0, a} is a left as well as right pseudo ideal of M and thus I is a pseudo ideal of M. This is also an ideal of M.

Note that left pseudo ideal and right pseudo ideal are independent concepts. Examples are given below

Example 6: Let (M, +) be an additive group such that M = {0, a, b, c} and Γ = {α, β} be a nonempty set of binary operations on M as shown in the following tables.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

α	0	a	b	c
0	0	0	0	0
a	0	a	0	0
b	0	b	0	0
c	0	c	0	0

β	0	a	b	c
0	0	0	0	0
a	0	0	0	0
b	0	0	0	0
c	0	0	0	0

Here I = {0, a} is a right pseudo ideal of M which is not a left pseudo ideal. Since bαa - bα0 =b $\notin$I.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

α	0	a	b	c
0	0	0	0	0
a	0	a	b	c
b	0	0	0	0
c	0	0	0	0

β	0	a	b	c
0	0	0	0	0
a	0	0	0	0
b	0	0	0	0
c	0	0	0	0

Here I = {0, a} is a left pseudo ideal of M which is not a right pseudo ideal. Since aαb =b $\notin$I.

Theorem 2.1.7:- Every left (resp. right) ideal of M is a left (resp. right) pseudo ideal of M but converse need not be true.

Proof: Let <M, +, α >, α∈Γ be a left ideal of M. Let I be a left pseudo ideal in M. So, <I, +> is a normal subgroup of <M, + > and by definition n.i – n.0 ∈ I for all i ∈ I and n ∈ M . Hence I is a left pseudo ideal of M.

Now we prove that I is a right pseudo ideal of M.

Let I be a right ideal of M. So, <I, +> is a normal subgroup of <M, + > and by definition iαn ∈ I for all i ∈ I, α ∈Γ and n ∈ M. Therefore I is right pseudo ideal of M.

Hence every left (right) ideal of M is a left (right) pseudo ideal of M but converse need not be true as shown in the following Example.

Example 7: Let (M, $+_4$) be an additive group modulo 4 where M = {0, 1, 2, 3} and Γ = {α, β} be a nonempty set of binary operations on M as shown in the following tables.

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

α	0	1	2	3
0	0	0	0	0
1	0	1	0	0
2	0	2	0	0
3	0	3	0	0

β	0	1	2	3
0	0	0	0	0
1	0	0	0	1
2	0	0	0	2
3	0	0	0	3

Then (M, +, Γ) is a Γ-near ring and I = {0, 2} is a pseudo ideal of M which is not an ideal. Since $3\alpha(1+2) - 3\alpha 1 = 3\alpha 3 - 3\alpha 1 = 0 - 3 = 3 \notin I$

In the following theorem we see that intersection of any family of left (resp. right) pseudo ideals of M is a left (resp. right) pseudo ideal of M respectively.

Theorem 2.1.8. Intersection of any collection of left (resp. right) pseudo ideals of M is a left (resp. right) pseudo ideal of M.

Proof: Let $I = \bigcap_{\lambda\in\Lambda} \{ I_\lambda / \lambda\in\Lambda$, where Λ is an index set} be a family of left pseudo ideals of M. Since $I_\lambda \neq \emptyset$ for all λ and $0 \in \bigcap_{\lambda\in\Lambda} I_\lambda = I$. So, $I \neq \emptyset$.

Also intersection of any collection of normal subgroups in M being normal. We get <I, +> is a normal subgroup in <M, +> . Let $i \in I$, where I_λ is a pseudo left ideals of M.

By definition of pseudo left ideals of M, $n\alpha i - n\alpha 0 \in I_\lambda$ for all I_λ, $\alpha \in \Gamma$. Hence , $n\alpha i - n\alpha 0 \in \bigcap_{\lambda\in\Lambda} I_\lambda$ for all I_λ , $\alpha \in \Gamma$ and for all $n \in M$. Thus $n\alpha i - n\alpha 0 \in I$ for all $i \in I$, $\alpha \in \Gamma$ and for all $n \in M$ i.e.I is a left pseudo ideal of M.

Now Let $I = \bigcap_{\lambda\in\Lambda} \{ I_\lambda / \lambda\in\Lambda$, where Λ is an index set} be a family of right pseudo ideals of M. Since $I_\lambda \neq \emptyset$ for all λ and $0 \in \bigcap_{\lambda\in\Lambda} I_\lambda = I$. So, $I \neq \emptyset$.

By definition of pseudo right ideals of M, $i\alpha n \in I_\lambda$ for all I_λ, $\alpha \in \Gamma$. Hence, $i\alpha n \in \bigcap_{\lambda\in\Lambda} I_\lambda$ for all I_λ , $\alpha \in \Gamma$ and for all $n \in M$. Thus $i\alpha n \in I$ for all $i \in I$, $\alpha \in \Gamma$ and for all $n \in M$ i.e. I is a right pseudo ideal of M.

REFERENCES

1. **G.Berman and R. J. Silverman,** "Near-rings", American Mathematical Monthly, 66 (1959),23-34.
2. **N. Nobusawa,** On a generalization of the Ring Theory, Osaka, J. Maths.1(1964) 81-89.
3. **Y.S.Pawar and V.B.Pandharpure,** Ideals and Bi-ideals in near-rings.News Bull.Cal.Math.Soc.27 (4-6) (2004) 5-12.
4. **G. Pilz,** Near Rings, North-Holland, Amsterdam, 1983.
5. **S.Pianskool and P.Khachorncharoenkul,** Simple Γ- semi-near rings, Journal of Mathematics Research, Vol. 1, No. 2 (2009),124-129.
6. **M. K. Rao,** Γ- semiring I, Southeast Asian Bulletin of Maths., 19(1995) 49-54.
7. **N. K. Saha,** On Γ- seminear ring II, Bull. Calcutta Math. Soc. 79(6) (1987), 331-335.
8. **Bh. Satyanarayan ,** A Note on Γ-Near-Ring , Indian Journal of Mathematics, B. N. Prasad Birth Centenary Commemoration , Vol.41, No. 3 (1990) ,427-433.
9. **M. K. Sen,** On pseudo-ideals of semi-group, Bull. Calcutta Math. Soc. 67 (1975), 109-114.
10. **S. K.Sen and N. K. Saha,** On Γ- seminear ring I, Bull. Calcutta Math. Soc., 78 (1986), no.3, 180-186.
11. **Wang Xue-Kuan**, On pseudoideals of division rings, Journal of Mathematical Research and Exposition (1988-02).

S. J. Alandkar
Head and Associate Professor, Department of Mathematics & Statistics,
Walchand College of Arts and Science, Solapur [M.S.]