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QUASI K-IDEALS IN K-REGULAR Γ −SEMIRINGS

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Abstract. A Γ-semiring (M, +, α), α∈ Γ whose additive reduct is a Γ-semilattice, is called a k-regular Γ -semiring if for each a ∈ M there exists x ∈ M, α ∈ Γ such that a + a$\alpha x \alpha$a = a$\alpha x \alpha$a,. Here we introduce quasi k-ideals in Γ-semirings and characterize the k-regular Γ-semirings by their quasi k-ideals.

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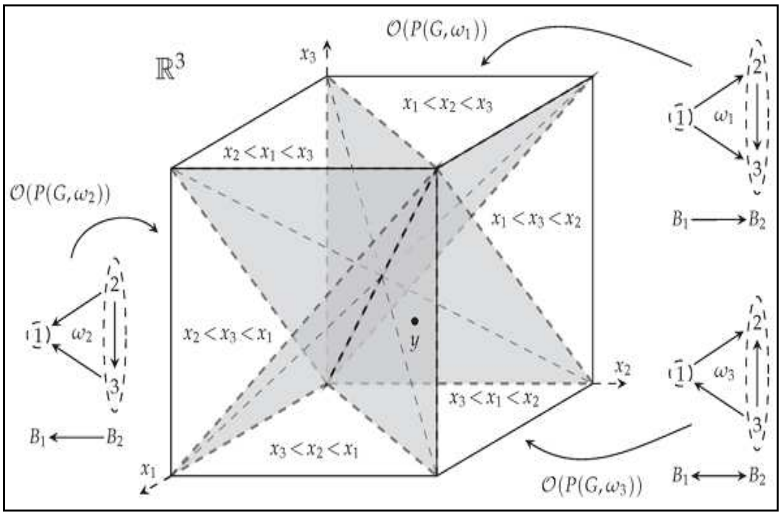
1 INTRODUCTION.

The concept of quasi ideals in rings and semigroups is studied by Otto Steinfeld [15], [16], [17], [18]. Quasi ideal is a particular case of bi-ideal. It is generalization of both left ideal and right ideal[19]. S. Lajos [9] discussed the generalization of this notion, namely the (m,n)-quasi ideal, He characterized the quasi ideals in regular semigroups [10]. Kapp [7] found that for an absorbant semigroup with 0 every $\mathcal{H}$-class together with 0 is a quasi ideal. Quasi ideals of different classes of semigroups and semirings have been characterized by many authors in [6], [8], [4].Von Neumann[11]defined a ring R to be regular if the multiplicative reduct (R, .) is a regular semigroup.



Rao M. K.[12] defined Γ-semiring as generalization of semiring and Γ-rings. Bhuniya at el [2] studyied ML+. In this article we introduce the notion of quasi k-ideals in a Γ-semiring and characterize the k-regular Γ-semirings using quasi k-ideals. Bourne [3] introduced the k-regular semirings as a generalization of regular rings. Later these semirings have been studied by Sen, Weinert, Bhuniya, Adhikari [1], [12], [13], [14]. For any semigroup F, the semiring P(F) of all subsets of F is a k-regular semiring if and only if F is a regular semigroup. Here we show that Q is a quasi k-ideal of the Γ-semiring P(F) if and only if Q = P(P) for some quasiideal P of F. Thus it is of interest to characterize the k-regular Γ-semirings using quasi k-ideals.In this paper we have discussed the characterization of the k-regular Γ -semirings by their quasi k-ideals.

2. PRELIMINARIES.

Definition 2.1. Γ- semiring:-Let (M, +) and **(** Γ, +) be commutative semigroups. Define the mapping M× Γ × M → M (image to be denoted by (x, α, y) →xαy) satisfying the following conditions:

i) xαy ∈ M,

ii) (x+y) αz = (xαz) + (yαz), x(α + β)z = xαz+ xβz, xα(y + z) = xαy+ xαz,

iii)(xαy)βz = xα(yβz), for all x, y, z ∈ M and all α, β∈ Γ.

Then M is a Γ- semiring.

Every Γ-ring is a Γ- semiring but every Γ-semiring need not be a Γ-ring. For this we consider the following Example.

Example 1. Let M= Γ = (Z^+, +) be the semigroup of all nonzero positive integers. Define the mapping M× Γ ×M → M (image to be denoted by (x, α, y) → xαy) where xαy is the usual multiplication of the x, α and y for all x, y ∈ M and all α∈ Γ. Then M is a Γ-semiring but not a semiring.

Definition 2.2. Sub-Γ-semiring:-Let M be a Γ-semiring. A nonempty subset S of M is a sub-Γ-semiring of M if S itself is a Γ-semiring with the same operations of Γ-semiring M.

Definition 2.3. Ideal of a Γ-semiring:-A nonempty subset I of a Γ-semiring M is a left (resp. right) ideal of M if for x, y ∈ I and r ∈M we have x+y ∈ I and rαx ∈I(resp. xαr ∈I), where α ∈ Γ. If I is both left as well as right ideal then we say that I is an ideal of M.

Example 2. Consider the Example of the Γ-semiring M followed by the definition. Here I = ($2Z^+$, +, Γ) is an ideal of M**.**

Following are the definitions introduced for generalization of algebraic structure semigroups, semilattice , semiring, reduct by considering set Γ for α ∈ Γ of operations by replacing multiplication operation (.):

Definition 2.4. A band is a Γ-semigroup in which every element is an idempotent. A commutative band is called a Γ-semilattice. Throughout this paper, unless otherwise stated, M is always a Γ-semiring whose additive reduct is a Γ-semilattice and the variety of all such Γ-semirings is denoted by ML+.

Definition 2.5. A non-empty subset L of a Γ-semiring M is called a left ideal of M if L + L⊆ L and M ΓL ⊆ L. The right ideals are defined dually. A subset I of M is called an ideal of M if it is both a left and a right ideal of M. A non-empty subset A is called an interior ideal of M if A + A ⊆ A and M ΓAΓ M ⊆ A. A non-empty subset A of M is called semiprime if for a ∈ M, a^2=aαa ∈ A implies that a ∈ A.

Definition 2.5. Henriksen [5] defined an ideal (left, right) I of a semiring S to be a k-ideal (left, right) if for a; x ∈ S, a; a + x ∈ I) x ∈ I. We extend to Γ-semiring M.

We define interior k-ideal similarly.

Definition 2.6. A non-empty subset A of M is called a k-subset of Γ-semiring M if for x ∈ M, a ∈ A; x + a ∈ A implies that x ∈ A.

Definition 2.8. The k-closure $\bar{A}$ of a non-empty subset A is given by,

$$\bar{A} = \{x \in S \,/\, \exists\ a, b \in A \text{ such that } x + a = b\}.$$

This is the smallest k-subset containing A. If A and B be two subsets of M such that A ⊆ B then it follows that $\bar{A} \subseteq \bar{B}$. Since the additive reduct (M, +) is a Γ-semilattice, it follows that an ideal (left, right) K of M is a k-ideal(left, right) if and only if $\bar{K}$ = K.

Definition 2.9. A sub Γ-semiring Q is called a quasi ideal of M if QΓM∩ MΓQ ⊆ Q. A quasi ideal Q is called a quasi k-ideal of M if $\bar{Q}$ = Q.

For examples of quasi k-ideals of a Γ-semiring we would like to explore the following natural connection between quasi ideals of a Γ-semigroup F and quasi k-ideals of the Γ-semiring P(F) of all subsets of F.

Definition 2.10. Let F be a Γ-semigroup and P(F) be the set of all subsets of F. Define addition and multiplication on P(F) by:

U + V = U ∪ V and

U Γ V = {aαb / a ∈ U; b ∈ V, α ∈ Γ} }, for all U, V ∈ P(F),

Then (P(F); +; α) , α ∈ Γ is a Γ-semiring whose additive reduct is a Γ-semilattice. Then we have the following result.

Theorem 2.11. Let F be a Γ-semigroup. Then Q is a quasi k-ideal of P(F) if and only if Q = P(P) for some quasi-ideal P of F.

Proof. Let P be a quasi ideal of F and Q = P(P). Let A = {a_1, a_2 , …, a_n} ∈ MΓQ ∩ Q Γ M where M = P(F). Then for each a_i there exist {s_i},{t_i} ∈ M and {p_i},{q_i} ∈ Q such that a_i = s_iα p_i = q_i α t_i. But a_i ∈ FΓP ∩ PΓF ⊆ P for all i. Thus A ⊆ P. Hence A ∈ Q. Therefore MΓQ ∩ QΓ M ⊆ Q. Thus Q is a quasi ideal of M. Now let U ∈ M and V_1,V_2 ∈ Q such that U + V_1 = V_2. Then we have U ∪ V_1 = V_2. Which implies that U ⊆P. Thus U ∈ Q. Therefore Q is a quasi k-ideal of M.

Conversely, let Q be a quasi k-ideal of M = P(F). We consider P = $\bigcup_{U \in Q} U$. Then P⊆F and Q ⊆ P(P). LetB ∈ P(P). Then B ∈ Q. Therefore Q = P(P).

We now prove that P is a quasi ideal of F i.e. FΓP ∩PΓF ⊆ P. Let x ∈ FΓP ∩PΓF. Then there exist p, q ∈ P and s, t ∈ F such that x = sαp = qαt. Now MΓQ ∩ QΓM ⊆ Q ⊆ P(F) = M. Now {s, t} {p, q } ∈ MΓQ and {p, q } {s, t} ∈ QΓ M. Also {sαp} = {s}α{p} ∈ MΓQ and {qαt} = {q} α {t} ∈ QΓM. Thus {x}∈ M ΓQ ∩QΓM ⊆ Q and so x∈P. Therefore F ΓP ∩ PΓF ⊆ P and hence P is a quasi ideal of F.

Lemma 2.12 Let M be a Γ-semiring . Then for all right k-ideal R and left k-ideal L of M, R ∩ L is a quasi k-ideal of M.

Proof. Let R and L be a right k-ideal and left k-ideal of M respectively. Then we have (R∩L) ΓM ∩ MΓ(R ∩ L) ⊆ RΓM ∩ MΓL as R∩L ⊆ R and R∩L ⊆ L ⊆ R∩L as RΓM ⊆ R and MΓR ⊆ L and so R ∩L is a quasi ideal of M. Since intersection of two k-subsets is a k-set of a Γ-semiring, it follows that R∩L is a quasi k-ideal of M. Let a∈M. We denote L[a] = $\{\sum_{i=1}^{n} x_i /\ x_i \in \{a\} \cup Ma\}$. Since the additive reduct (M, +) is a semilattice, it follows that L[a] = {a + sαa / s∈ M, α ∈ Γ }. Then L[a] is a sub-Γ-semiring of M. Also for any s ∈ M and u ∈ L[a], we have sαu ∈ Ma which implies that MΓL[a] ⊆ L[a] and so L[a] is a left ideal of M. As in [2], following description for the principal left k-ideal $L_k(a)$ and right k-ideal $R_k(a)$ of M can be verified easily.

Lemma 2.13. Let M be a Γ-semiring and a ∈ M.
1.Then the principal left k-ideal of M generated by a is given by $L_k(a)$ = {u ∈ M / u+a + sαa = a + sαa, for some s∈ M, α ∈ Γ }.
2.Then the principal right k-ideal of M generated by a is given by $R_k(a)$ = {u ∈M / u + a + aαs = a + aαs,for some s ∈ M, α ∈ Γ }.

3. QUASI IDEALS IN K-REGULAR Γ-SEMIRINGS.

Bourne [3] defined a Γ-semiring M to be regular if for each a∈ M there exist x, y∈ M such that a + aαxαa = aαyαa, for α ∈ Γ. If a Γ-semiring M happens to be a ring then the Von Neumann regularity and the Bourne regularity are equivalent. This is not true in a Γ-semiring in general (For counter example we refer [12]). Adhikari, Sen and Weinert [1] renamed the Bourne regularity of a Γ-semiring as k-regularity to distinguish from the notion of Von Neumann regularity.

Definition 3.1 A Γ-semiring M is called a k-regular Γ-semiring if for each a ∈ M there exist x, y ∈ M such that a + aαxαa = aαyαa, α ∈ Γ.

Since (M, +) is a semilattice,

we have a + aαxαa = aαyαa ⟹ a + aαxαa + (aαxαa + aαyαa) = aαyαa + (aαxαa +aαyαa)

⟹ a + aα(x+y)αa = aα(x+ y)αa .

Thus, a Γ-semiring M is k-regular if and only if for all a∈ M there exists x∈ M, α ∈ Γ such that a + aαxαa = aαxαa..

Let M be a k-regular Γ-semiring and a∈ M. Then there exists x∈M, α ∈ Γ such that a + aαxαa = aαxαa. Then we have

a + aαxαa = aαxαa ⟹ a + aαxα(a + aαxαa) = aαxα(a + aαxαa)

⟹ a + aαxαaαxαa = aαxαaαxαa.

Thus, a Γ-semiring M is k-regular if and only if for all a ∈ M there exists x ∈ M such that

a + aαxαaαxαa = aαxαaαxαa … (1)

For examples and properties of k-regular Γ-semirings we refer [1], [12], [13], [14].
We observe that the proof of this result can be made signicantly simpler when the Γ-semiring M is taken from ML+.

Theorem 3.2 Let M be a Γ-semiring . Then M is k-regular if and only if $\overline{R\Gamma L}$ = R∩L for any right k-ideal R and left k-ideal L of M.

Proof. Let M be a k-regular Γ-semiring . Then for any right k-ideal R and left k-ideal L of M, R ΓL ⊆ RΓM ⊆ R and RΓL ⊆ MΓL ⊆ L. Then $\overline{R\Gamma L}$ ⊆ R∩L implies that RΓL ⊆ R ∩ L. Also for a ∈ R∩L there exists x∈ M such that a + aαxαa = aαxαa. Then (aαx)αa ∈ RΓL implies that a ∈ $\overline{R\Gamma L}$ and so R∩L ⊆ $\overline{R\Gamma L}$. Thus $\overline{R\Gamma L}$ = R∩L.

Conversely, let $\overline{R\Gamma L}$ = R∩L for any right k-ideal R and left k-ideal L of M. Let a ∈ M, α ∈ Γ, R = $R_k(a)$ = {u ∈ M / u + a + aαs = a + aαs} and L = $L_k(a)$ = {v ∈ M / v +a+ sαa = a + sαa}. Then a∈ R ∩ L = $\overline{R\Gamma L}$. Then there exist u ∈ R and v ∈ L such that a + uαv = uαv. This implies that a + (a + aαs)α(sαa + a) = (a + aαs)(sαa + a). Thus a + aαyαa = aαyαa for some y∈ M, α ∈ Γ. Hence M is k-regular. Now we give several equivalent characterizations of k-regularity in terms of quasi k-ideals.

Theorem 3.3 Let A be a non-empty subset of Mand A be a k-ideal of M if and only if k-regular Γ-semiring . Then A is a quasi A = $\overline{R\Gamma L}$, where R is a right k-ideal and L is a left k-ideal of M.

Proof. Let A be a quasi k-ideal of k-regular Γ-semiring M and a∈ A. Then R = $R_k(a)$ and L = $L_k(a)$ are right k-ideal and left k-ideal of M respectively. Since M is k-regular and a∈A ⊆ M, there is x ∈ M such that a + aαxαaαxαa = aαxαaαxαa. Now a∈R, α ∈ Γ ⟹ aαx ∈ R and a ∈ L ⟹(aαx)αa ∈ L. Then aαxαaαxαa ∈ RΓL. Thus a ∈ $\overline{R\Gamma L}$. Therefore A ⊆ $\overline{R\Gamma L}$. Now consider u ∈ R, α ∈ Γ and v∈L. Then, by Lemma 2.4, there are s, t ∈ M such that u + aαs + a = aαs + a and v + tαa + a = tαa + a. Then uαv + (aαs + a) αv = (aαs + a) αv implies that uαv + (aαs + a)α(tαa + a) = (aαs + a)α(tαa + a) ⟹uαv + a α(s α t + s + t)αa + a^2 = aα(sαt + s + t)αa + a^2. Again aα(sαt + s + t)αa + a^2 ∈ AΓ M ∩ MΓA ⊆ A shows that uαv ∈ $\overline{A}$ ⊆A. Then RΓL ⊆ A and so $\overline{R\Gamma L}$ ⊆ A. Hence A = $\overline{R\Gamma L}$.

Conversely, let for a non-empty subset A of M, A = $\overline{R\Gamma L}$, where R is a right k- ideal and L is a left k- ideal of M. But by the above theorem, for a k-regular Γ-semiring M, $\overline{R\Gamma L}$ = R ∩ L. Then A = R ∩L. But R ∩L is quasi k-ideal of S[Lemma 2:3]. Thus A is a quasi k-ideal of M.

Theorem 3.4 For a Γ-semiring M the following conditions are equivalent:
1. M is k-regular.
2. Q =$\overline{Q\Gamma M\Gamma Q}$ for every quasi k-ideal Q of M.

Proof. (1) ⟹ (2): Let Q be a quasi k-ideal of M. Then QΓMΓQ ⊆ QΓM∩MΓQ ⊆ Q implies that $\overline{Q\Gamma M\Gamma Q}$ ⊆ Q. Let a ∈ Q. Since M is k-regular, there is x ∈M such that a + aαxαa = aαxαa. Then aαxαa ∈ QΓMΓQ implies that a ∈ $\overline{Q\Gamma M\Gamma Q}$, whence Q ⊆ $\overline{Q\Gamma M\Gamma Q}$. Thus Q = $\overline{Q\Gamma M\Gamma Q}$.
(2) ⟹ (1) : Let a ∈ M. Then Q = $L_k(a)$ ∩ $R_k(a)$ is a quasi k-ideal of M, by Lemma 2.3 and 2.4. Then a ∈ Q = QΓMΓQ and this implies that there exist q_1, q_2, q_3, q_4 ∈ Q and s_1, s_2 ∈ M, α ∈ Γ such that
$a + q_1\alpha s_1\alpha q_2 = q_3\alpha s_2\alpha q_4$

$\Longrightarrow a + (q_1 + q_2 + q_3 + q_4)\,\alpha\,(s_1 + s_2)\,\alpha\,(q_1 + q_2 + q_3 + q_4) = (q_1 + q_2 + q_3 + q_4)\,\alpha\,(s_1 + s_2)\,\alpha\,(q_1 + q_2 + q_3 + q_4)$
$\Longrightarrow a + q\,\alpha\,s\,\alpha\,q = q\alpha s\alpha q$, where $q = (q_1 + q_2 + q_3 + q_4) \in Q = L_k(a) \cap R_k(a)$ and $s = (s_1 + s_2) \in M$. Then there exist x, y $\in M$, $\alpha \in \Gamma$ such that $q + x\alpha a + a = x\alpha a + a$ and $q + a\alpha y + a = a\alpha y + a$[Lemma 2.4]. Thus we have
$a + q\,\alpha s\alpha q = q\alpha s\alpha q$
$\Longrightarrow a + (q + a\alpha y + a)\alpha\, s\alpha(q + x\alpha a + a) = (q + a\alpha y + a)\,\alpha s\,\alpha\,(q + x\,\alpha\,a + a)$
$\Longrightarrow a + (a\alpha y + a)\,\alpha\,s\,\alpha\,(x\,\alpha\,a + a) = (a\,\alpha\,y + a)\,\alpha\,s\,\alpha\,(x\,\alpha\,a + a)$
$\Longrightarrow a + a\,\alpha\,(y\,\alpha\,s\,\alpha\,x + y\,\alpha\,s + s\,\alpha\,x + s)\,\alpha\,a = a\,\alpha\,(y\,\alpha\,s\,\alpha\,x + y\,\alpha\,s + s\,\alpha\,x + s)\,\alpha\,a$
$\Longrightarrow a + a\,\alpha\,t\,\alpha\,a = a\,\alpha\,t\,\alpha\,a$, where $t = y\,\alpha\,s\,\alpha\,x + y\,\alpha\,s + s\,\alpha\,x + s \in M$, $\alpha \in \Gamma$ and so M is a k-regular Γ-semiring .

Theorem 3.5 For a Γ-semiring M the following conditions are equivalent:
1. M is k-regular.
2.$Q \cap J = \overline{Q\Gamma J\Gamma Q}$ for every quasi k-ideal Q and every k-ideal J of M.
3.$Q \cap I = \overline{Q\Gamma I\Gamma Q}$ Q for every quasi k-ideal Q and every interior k-ideal I of M.
Proof. Since each k-ideal is an interior k-ideal, it is clear that (3) ⟹ (2). Hence we are to prove (1) ⟹ (3) and (2) ⟹ (1) only.
(1) ⟹ (3): Let Q be a quasi k-ideal and I be an interior k-ideal of M. Then $Q\Gamma I\Gamma Q \subseteq Q\Gamma M\Gamma Q \subseteq Q\Gamma M \cap M\Gamma Q \subseteq Q$ and $Q\Gamma I\Gamma Q \subseteq M\Gamma I\Gamma M \subseteq I$ implies that $Q\Gamma I\Gamma Q \subseteq Q \cap I$ and so $\overline{Q\Gamma I\Gamma Q} \subseteq Q \cap I$. Let $a \in Q\cap I$. Since M is k-regular, there is $x \in M$ such that $a + a\alpha\, x\alpha\, a\alpha x\alpha a = axaxa$, $\alpha \in \Gamma$ by (1). Now $a\alpha(x\alpha a\alpha x)\,\alpha\,a \in Q\Gamma\,(M\Gamma I\,\Gamma M)\,\Gamma Q \subseteq Q\Gamma I\Gamma Q$ implies that $a \in Q\Gamma I\Gamma Q$ and so $Q \cap I \subseteq \overline{Q\Gamma I\Gamma Q}$. Thus $Q \cap I = \overline{Q\Gamma I\Gamma Q}$.
(2) ⟹ (1): Let Q be a quasi k-ideal of M. Since M is a k-ideal of M, $Q\cap M = \overline{Q\Gamma M\Gamma Q}$ i.e. $Q = \overline{Q\Gamma M\Gamma Q}$.
Hence M is a k-regular Γ-semiring , by Theorem 3.4.
Theorem 3.6 For a Γ-semiring M the following conditions are equivalent:
1. M is k-regular.
2. $R\cap L \subseteq \overline{R\Gamma L}$ for every right k-ideal R and every left k-ideal L of M.
3. $Q\cap L \subseteq \overline{Q\Gamma L}$ for every quasi k-ideal Q and every left k-ideal L of M.
Proof. (1) ⟹ (3) : Let Q be a quasi k-ideal and L be a left k-ideal of M respectively. Let $a\in Q\cap L$. Since M is k-regular, there is $x\in M$, $\alpha \in \Gamma$ such that $a+a\alpha x\alpha a = a\alpha x\alpha a$. Now $a\alpha(x\alpha a) \in Q\,\Gamma(M\Gamma L) \subseteq Q\Gamma L$. Then $a\in Q\Gamma L$. Thus $Q\cap L \subseteq Q\Gamma L$.
(3) ⟹ (2) : Since every right k-ideal is a quasi k-ideal of M, it follows that $R \cap L \subseteq \overline{R\Gamma L}$.
(2) ⟹ (1) : Let $a \in M$. Consider $L = L_k(a)$, $R = R_k(a)$. Then $a \in R \cap L$ implies that there exist $r \in R$ and $l \in L$ such that $a + r\,\alpha\,l = r\,\alpha\,l$. Again there exist $s, t \in M$ such that $r + a\,\alpha\,s + a = a\,\alpha\,s + a$ and
$l + t\,\alpha\,a + a = t\,\alpha\,a + a$. Thus we have
$a + r\,\alpha\,l = r\,\alpha\,l \Longrightarrow a + (r + a\,\alpha\,s + a)\,\alpha\,(l + t\,\alpha\,a + a) = (r + a\,\alpha\,s + a)\,\alpha\,(l + t\,\alpha\,a + a)$
$\Longrightarrow a + a^2 + a\,\alpha\,u\,\alpha\,a = a^2 + a\,\alpha\,u\,\alpha\,a$, for some $u \in M$
$\Longrightarrow a + a\,\alpha\,(a + a^2 + a\,\alpha\,u\,\alpha\,a) + a\,\alpha\,u\,\alpha\,a = a\,\alpha\,(a + a^2 + a\,\alpha\,u\,\alpha\,a) + a\,\alpha\,u\,\alpha\,a$
$\Longrightarrow a + a\alpha\,v\,\alpha a = a\alpha v\alpha a$, for some $v = a + a\,\alpha\,u + u \in M$, whence M is k-regular Γ-semiring .
The left-right dual of this theorem is as follows:
Theorem 3.7 For a Γ-semiring M the following conditions are equivalent:
1. M is k-regular.
2.$Q \cap R \subseteq \overline{R\Gamma Q}$ for every quasi k-ideal Q and every right k-ideal R of M.
Theorem 3.8 For a Γ-semiring M, the following conditions are equivalent:
1. M is k-regular.
2.$R \cap Q \cap L \subseteq \overline{R\Gamma Q\Gamma L}$ for every right k-ideal R, every quasi k-ideal Q and every left k-ideal L of M.
Proof. (1) ⟹ (2): Let R, Q and L be any right k-ideal, any quasi k-ideal and any left k-ideal of M respectively. Let $a\in R \cap Q\cap L$. Since M is k-regular there exists $x \in M$, $\alpha \in \Gamma$ such that $a + a\alpha x\alpha a\alpha x\alpha a = a\alpha x\alpha a\alpha x\alpha a$. However $(a\alpha x)\,\alpha a\alpha(x\alpha a)\in R\Gamma Q\Gamma L$, whence $a \in \overline{R\Gamma Q\Gamma L}$. Thus $R\cap Q\cap L \subseteq \overline{R\Gamma Q\Gamma L}$.
(2) ⟹ (1): Let R and L be any right k-ideal and any left k-ideal of M respectively. Then $R\cap L$ is quasi k-ideal of M. Then we have
$R\cap(R\cap L)\cap L \subseteq \overline{R\Gamma(R\Gamma L)\Gamma L} \Longrightarrow R \cap L \subseteq \overline{R\Gamma L}$ and so M is a k-regular Γ-semiring , by Theorem 3.6.

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