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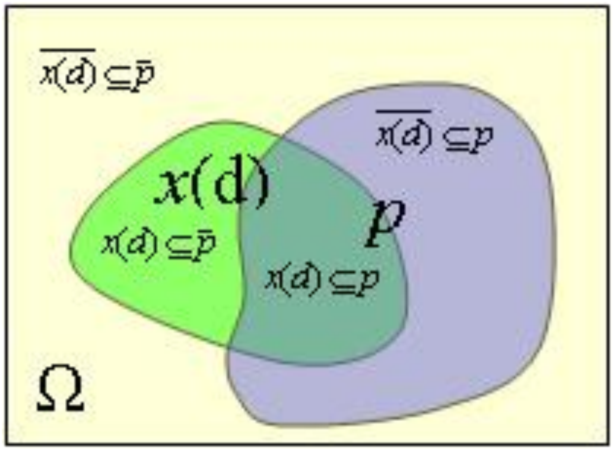
SOME PARAMETERS DOMINATION OF THE INDEPENDENT INTUITIONISTIC FUZZY GRAPH



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ABSTRACT

A non –empty set D $\subseteq$ V of a graph G is a dominating set of G if every vertex in V-D is adjacent to some vertex in D. The domination number γ(G) is the minimum cardinality taken over all the minimal dominating sets of G. If V-D contains a dominating set D^1 then D^1 is called the Inverse dominating set of G w.r.to D. The Inverse dominating number γ^1(G) is the minimum cardinality taken over all the minimal inverse dominating sets of G. An independent dominating set D of a IFG G=(V,E) is a split independent dominating set if the induced fuzzy subgraph$< V\text{-}D >$ is disconnected .The minimum fuzzy cardinality of a split independent dominating set is called a split independent domination number and is denoted by $\gamma_{spiif}(G)$.

Key Words: Split Independent Dominating Set**,** Intuitionistic Fuzzy Graph, Independent Strong (weak) Dominating Set, Efficient Independent Dominating Set.

Introduction

A non –empty set D $\subseteq$ V of a graph G is a dominating set of G if every vertex in V-D is adjacent to some vertex in D. The domination number (G) is the minimum cardinality taken over all the minimal dominating sets of G. If V-D contains a dominating set D^1 then D^1 is called the Inverse dominating set of G w.r.to D. The Inverse dominating number γ^1(G) is the minimum cardinality taken over all the minimal inverse dominating sets of G. Two vertices in an IFG, G=(V,E) are said to be independent if there is no strong arc between them. A subset S of V is said to be Independent set of G if $\mu_2(u,v)<\mu_2^{\infty}(u,v)$ and $\gamma_2(u,v)<\gamma_2^{\infty}(u,v)$ for all u,v ϵ S. A dominating set D of a fuzzy graph G= (V, E) is a split dominating set if the induced fuzzy subgraph H = (<V-D>,V^1,E^1) is disconnected. The split domination number $\gamma_2(G)$ of G is the minimum fuzzy cardinality of a split dominating set. A dominating set D of a Intuitionistic fuzzy graph G=(V,E) in a split dominating set if the induced fuzzy sub graph H=(<V-D>,V^1,E^1)is disconnected . The minimum fuzzy cardinality of a split dominating set is called a split domination number and is denoted by $\gamma_2(G)$. An arc (vi,vj) of an IFG G is called an strong arc if $\mu_2(vi,vj) \le \mu_1(vi) \wedge \mu_1(vj)$ and $\gamma_2(vi,vj) \le \gamma_1(vi)\wedge\gamma_1(vj)$. Let G=(V,E) be a IFG. Then the cardinality of G is defined to be $|G| = |\sum_{vi\in V}[(1+\mu_1(vi)-\gamma_1(vi))/2]+ \sum_{vi\in V}[(1+\mu_2(vi,vj)-\gamma_2(vi,vj))/2]|$. The vertex cardinality of G is defined by $|V| =\sum_{vi\in V}[(1+\mu_1(vi)-\gamma_1(vi))/2]$ for all vi ϵ V. The edge cardinality of G is defined by $|E| =\sum_{vi\in V}[(1+\mu_2(vi,vj)-\gamma_2(vi,vj))/2]$ for all (vi,vj)ϵ E. The vertex cardinality of an IFG is called the order of G and is denoted by O(G) . The cardinality of the edges in G is called the size of G, it is denoted by S(G).

Split Independent Dominating Set in IFG

An independent dominating set D of a IFG G=(V,E) is a split independent dominating set if the induced fuzzy subgraph< V-D > is disconnected .The minimum fuzzy cardinality of a split independent dominating set is called a split independent domination number and is denoted by $\gamma_{spiif}(G)$.

Example

Let G=(V,E) be a fuzzy graph with V={a, b, c, d, e}, the membership functions of vertices and edges are given below

$(\mu_1(a), \gamma_1(a)) = (0.3, 0.7)$, $(\mu_1(b), \gamma_1(b)) = (0.3, 0.6)$
$(\mu_1(c), \gamma_1(c)) = (0.3, 0.4)$, $(\mu_1(d), \gamma_1(d)) = (0.6, 0.4)$
$(\mu_1(e), \gamma_1(e) = (0.7,0.3)$ and
$(\mu_2(ab), \gamma_2(ab)) = (0.2, 0.4)$, $(\mu_2(ac), \gamma_2(ac)) = (0.3, 0.7)$
$(\mu_2(ad), \gamma_2(ad)) = (0.2, 0.4)$, $(\mu_2(bc), \gamma_2(bc)) = (0.3, 0.5)$
$(\mu_2(bd), \gamma_2(bd)) = (0.3, 0.4)$, $(\mu_2(ce), \gamma_2(ce)) = (0.2, 0.4)$
$(\mu_2(de), \gamma_2(de)) = (0.6, 0.4)$.

Here strong arcs are e_1,e_2,e_5 and e_4

Independent dominating set in IFG is D=(a, b, e}, V-D = {c , d} For every v ϵV-D their exists u ϵ D and V-D is induced intuitionistic fuzzy subgraph and it is independent and disconnected. That is two isolated vertices c and d. The minimum intuitionistic fuzzy cardinality of a split independent dominating set is called split independent domination number $\gamma_{spiif}(G)$ – 1.35

Independent Strong (Weak) Dominating Set in IFG

A SIFD – set (WIFD-set) S of an IFG G is said to be an independent strong (weak) dominating set of G if it is independent .The minimum cardinality of an independent strong (weak) dominating set is called the independent strong (weak) intuitionistic fuzzy dominating number and it is denoted by $\gamma_{isif}(G)$, $\gamma_{iwif}(G)$.

Example

Let G=(V,E) be a fuzzy graph with V={a, b, c, d, e}, the membership functions of vertices and edges are given below

$(\mu_1(a), \gamma_1(a)) = (0.4, 0.5)$, $(\mu_1(b), \gamma_1(b)) = (0.4, 0.6)$
$(\mu_1(c), \gamma_1(c)) = (0.2, 0.6)$, $(\mu_1(d), \gamma_1(d)) = (0.5, 0.6)$
$(\mu_1(e), \gamma_1(e) = (0.3,0.4)$, $(\mu_1(f), \gamma_1(f)) = (0.2, 0.7)$
$(\mu_1(g), \gamma_1(g)) = (0.3, 0.6)$ and
$(\mu_2(ab), \gamma_2(ab)) = (0.4, 0.5)$, $(\mu_2(bc), \gamma_2(bc)) = (0.2, 0.6)$
$(\mu_2(bd), \gamma_2(bd)) = (0.4, 0.6)$, $(\mu_2(cd), \gamma_2(cd)) = (0.2, 0.4)$
$(\mu_2(de), \gamma_2(de)) = (0.3, 0.4)$, $(\mu_2(ef), \gamma_2(ef)) = (0.2, 0.6)$
$(\mu_2(fg), \gamma_2(fg)) = (0.2, 0.7)$.

For an IFG in example $\gamma_{isif}(G)$- 0.65
Since{b,f} is a independent strong dominating set.

Efficient Independent Dominating Set in IFG

Let G=(V,E) be a IFG. A set F $\subseteq$ V is an efficient independent dominating set if F is independent dominating set and if for every v є V-F then N[v]$\cap$ F=1.
The efficient independent intuitionstic fuzzy domination number is the minimum cardinality among all efficient independent domination set in G and is denoted by $\gamma_{eiif}(G)$.

Example

Let G=(V,E) be a fuzzy graph with V={a, b, c, d, e}, the membership functions of vertices and edges are given below

$(\mu_1(a), \gamma_1(a)) = (0.2, 0.7)$, $(\mu_1(b), \gamma_1(b)) = (0.3, 0.6)$
$(\mu_1(c), \gamma_1(c)) = (0.4, 0.4)$, $(\mu_1(d), \gamma_1(d)) = (0.6, 0.4)$
$(\mu_1(e), \gamma_1(e) = (0.7,0.1)$, and
$(\mu_2(ab), \gamma_2(ab)) = (0.2, 0.7)$, $(\mu_2(ac), \gamma_2(ac)) = (0.2, 0.7)$
$(\mu_2(ad), \gamma_2(ad)) = (0.2, 0.7)$, $(\mu_2(bc), \gamma_2(bc)) = (0.3, 0.6)$
$(\mu_2(bd), \gamma_2(bd)) = (0.3, 0.6)$, $(\mu_2(ce), \gamma_2(ce)) = (0.4, 0.4)$
$(\mu_2(de), \gamma_2(de)) = (0.5, 0.4)$.

F={b,g,h}
$\gamma_{eiif}(G)$=1.45

Inverse dominating set in IFG

Let D be a minimum dominating set of an IFG of G if V-D contains a dominating set w.r.to $D^{'}$ of G. then $D^{'}$ is called an inverse dominating set w.r to D, The inverse domination number of an IFG G is the minimum cardinality of an inverse dominating set and it is denoted by $\gamma_{if}^{1}(G)$

Inverse independent dominating set in IFG

Let D $\subseteq$ V be a minimum independent dominating set of an IFG of G if V-D contains an independent dominating set D^1 of G then D^1 is called an inverse independent dominating set w.r.to D. The inverse independent domination number of an IFG G is the minimum cardinality of an inverse independent dominating set and it is denoted by$\gamma_{iif}^{1}(G)$

Example

Let G=(V,E) be a fuzzy graph with V={a, b, c, d, e}, the membership functions of vertices and edges are given below

$(\mu_1(a), \gamma_1(a)) = (0.3, 0.4)$, $(\mu_1(b), \gamma_1(b)) = (0.4, 0.4)$
$(\mu_1(c), \gamma_1(c)) = (0.5, 0.2)$, $(\mu_1(d), \gamma_1(d)) = (0.7, 0.2)$
$(\mu_1(e), \gamma_1(e)) = (0.3, 0.5)$, $(\mu_1(f), \gamma_1(f)) = (0.3, 0.4)$
$(\mu_1(g), \gamma_1(g)) = (0.3, 0.4)$ and
$(\mu_2(ad), \gamma_2(ad)) = (0.3, 0.7)$, $(\mu_2(bd), \gamma_2(bd)) = (0.4, 0.4)$
$(\mu_2(cd), \gamma_2(cd)) = (0.4, 0.2)$, $(\mu_2(de), \gamma_2(de)) = (0.3, 0.5)$
$(\mu_2(bd), \gamma_2(bd)) = (0.3, 0.6)$, $(\mu_2(ef), \gamma_2(ef)) = (0.2, 0.4)$
$(\mu_2(eg), \gamma_2(eg)) = (0.2, 0.5)$,
D^1={a,b,c,e}
$\gamma_{iif}^{1}(G)=2$

Results

Theorem

Let G be a IFG, then $\gamma_{isif}(G) \leq \gamma_{iwif}(G)$

Proof

Let S,W be minimal strong and weak dominating set respectively.

Let $d_N(u)=\Delta_N(G)$ & $d_N(v) = \delta_N(G)$ note that V-N(u) is a strong dominating set & V-N(v) is a weak dominating set of G

$\gamma_{isif}(G) \leq |V-N(u)|_{if} = \gamma_{isif}(G) \leq O(G)-\Delta_N(G)$ -----------(1)

& $\gamma_{iwif}(G) \leq |V-N(v)|_{if} = O(G)-\delta_N(G)$-----------(2)

We know that $O(G)-\Delta_N(G) \leq O(G)-\delta_N(G)$

Using (1) & (2) we get

$\gamma_{isif}(G) \leq \gamma_{iwif}(G)$

Theorem

For any graph IFG, $\gamma^{-1}(G) \leq \beta_0(G)$

Proof

Let D be a minimum dominating set of G. Let S be a maximal independent set in <V-D>.we now consider the following two case

Case(i):

Suppose $V - D - S = \phi$

Then V- D = S is an independent inverse dominating set of G

Thus$\gamma^{-1}(G) \leq |V-D| = |S| \leq \beta_0(G)$

Case(ii):

Suppose $V - D - S \neq \phi$

Then every in V – D – S is adjacent to at least one vertex in S

If every vertex in D is adjacent to atleast one vertex is S, then S is an inverse dominating set of G.

Otherwise, let $D^1 \subset D$ be a set of vertices of S. since D is a minimum dominating set, every vertex in D^1 must be atleast one vertex in V- D -S . Let $S^1 \subset V - D - S$ be such that every vertex of D^1 is adjacent to at least one vertex in S^1 ,clearly $|S^1| \leq |D^1|$ and $S \cup S^1$ is an inverse dominating set ,

Thus, $\gamma^{-1}(G) \leq | S \cup S^1 | \leq | S \cup D^1 | \leq \beta_0(G)$.

Conclusion

In this paper we have introduce the concept of Split independent dominating set in Intuitionistic fuzzy graph, Independent strong (weak) dominating set in Intuitionistic fuzzy graph , Inverse dominating set in Intuitionistic fuzzy graph , Inverse independent dominating set in Intuitionistic fuzzy graph .Some interesting results related with the above are proved . Further, the authors proposed to introduce new dominating parameters in Intuitionistic fuzzy graph and apply these concepts to Intuitionistic fuzzy graph models.

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