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PLANE WAVE SOLUTIONS OF FIELD EQUATIONS $R_{ij}=0$ IN V_5 WITH THREE TIME AXES

$$\left(\gamma_\mu \frac{\partial}{\partial x_\mu} + \frac{mc}{\hbar}\right)\psi = 0$$

$$\psi_{\vec{p}}(x) = u_{\vec{p}} e^{i(p_\rho x_\rho)/\hbar}$$

$$\left(\gamma_\mu \frac{ip_\mu}{\hbar} + \frac{mc}{\hbar}\right)\psi = 0$$

$$\gamma_i = \begin{pmatrix} 0 & -i\sigma_i \\ i\sigma_i & 0 \end{pmatrix}$$

$$\gamma_4 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

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Abstract:- The plane wave solutions of the field equations $R_{ij}=0$ five dimensional Space-time V_5 with three time axes for general theory of relativity are given by g_{ij} satisfying

$$N\rho_{\alpha\beta} + M\sigma_{\alpha\beta} = 0, \quad \alpha,\beta = 1,2.3,4,5.$$

Which further breaks into

$$\overset{=}{w}\rho_{\alpha\beta} + \overset{-}{w}\sigma_{\alpha\beta} = 0,$$

$$\overset{=}{\phi}_1\rho_{\alpha\beta} + \overset{-}{\phi}_1\sigma_{\alpha\beta} = 0,$$

$$\overset{=}{\phi}_2\rho_{\alpha\beta} + \overset{-}{\phi}_2\sigma_{\alpha\beta} = 0,$$

$$\overset{=}{\phi}_3\rho_{\alpha\beta} + \overset{-}{\phi}_3\sigma_{\alpha\beta} = 0,$$

$$\overset{=}{\phi}_4\rho_{\alpha\beta} + \overset{-}{\phi}_4\sigma_{\alpha\beta} = 0$$

where $\phi_1 = \frac{Z,_1}{Z,_5}$, $\phi_2 = \frac{Z,_2}{Z,_5}$, $\phi_3 = \frac{Z,_3}{Z,_5}$, $\phi_4 = \frac{Z,_4}{Z,_5}$

$$w = \phi_\alpha x^\alpha = \phi_1 y + \phi_2 z + \phi_3 t_1 + \phi_4 t_2 + t_3$$

$$\rho_{\alpha\beta} = -\phi_\alpha \phi_\beta L_2 + \frac{1}{2}[\phi_\alpha \rho_\beta + \phi_\beta \rho_\alpha],$$

$$\sigma_{\alpha\beta} = -\bar{\rho}_{\alpha\beta} + \frac{1}{4}[\phi_\alpha \phi_\beta L_1 - 2L_2(\phi_\alpha \rho_\beta + \phi_\beta \rho_\alpha) + 2\rho_\alpha \rho_\beta]$$

If Z is independent of the variable y the work regarding the plane wave Solutions in five dimensional space-time V_5 *having three time exes demonstrated in the paper refer it to Thengane (2003) can be deduced.*

Keywords: Plane Wave , Field Equations , dimensional .

INTRODUCTION:

In the paper referred it to Thengane (2003) ,he has obtained the plane wave Solutions g_{ij} of the field equations R_{ij} =0 in five dimensional space-times V_5 *having three time axes by reformulating Takeno's(1961) definition of plane wave as follows :*

Definition A plane wave g_{ij} is a non-flat solution of the field equations

$$R_{ij} = 0, \quad (i, j = 1,2,3,4,5) \tag{1.1}$$

in an empty region of the space-times such that

$$g_{ij} = g_{ij}(Z), \quad Z = Z(x^i), \quad x^i = y, z, t_1, t_2, t_3 \tag{1.2}$$

in some suitable co-ordinate system such that (1.3)

$$g^{ij} Z,_i Z,_j = 0, \quad Z,_i = \frac{\partial Z}{\partial x^i},$$

$$Z = Z(y, z, t_1, t_2, t_3) \quad Z,_1 \neq 0, \; Z,_2 \neq 0, \; Z,_3 \neq 0, \; Z,_4 \neq 0, \; Z,_5 \neq 0 \tag{1.4}$$

In this definition, the signature convention adopted is

$$g_{rr} < 0, \; r = 1,2$$

$$\begin{vmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{vmatrix} > 0, \quad g_{33} > 0, \quad g_{44} > 0, \; g_{55} > 0 \tag{1.5}$$

[not summed for $r = 1,2]$ and accordingly $g = \det(g_{ij}) > 0$. (1.6)

The field equations R_{ij} =0 then yield

$$\rho_i = \bar{g}_{ij} w^j = 0$$

$$N\rho_{\alpha\beta} + M\sigma_{\alpha\beta} = 0, \quad \alpha, \beta = 2,3,4,5.$$

Which further breaks into

$$\bar{\bar{w}}\rho_{\alpha\beta} + \bar{w}\sigma_{\alpha\beta} = 0,$$

$$\bar{\bar{\phi}}_2\rho_{\alpha\beta} + \bar{\phi}_2\sigma_{\alpha\beta} = 0,$$

$$\bar{\bar{\phi}}_3\rho_{\alpha\beta} + \bar{\phi}_3\sigma_{\alpha\beta} = 0,$$

$$\bar{\bar{\phi}}_4\rho_{\alpha\beta} + \bar{\phi}_4\sigma_{\alpha\beta} = 0$$

Where

$$w = \phi_\alpha x^\alpha = \phi_2 z + \phi_3 t_1 + \phi_4 t_2 + t_3$$

$$Z,_2 = \frac{\phi_2}{M}, \quad Z,_3 = \frac{\phi_3}{M}, \quad Z,_4 = \frac{\phi_4}{M}, \quad Z,_5 = \frac{1}{M}$$

$$\phi_2 = \frac{Z,_2}{Z,_5}, \quad \phi_3 = \frac{Z,_3}{Z,_5}, \quad \phi_4 = \frac{Z,_4}{Z,_5}$$

$$M = \bar{w} - (\bar{\phi}_2 z + \bar{\phi}_3 t_1 + \bar{\phi}_4 t_2)$$

$$N = \bar{\bar{w}} - (\bar{\bar{\phi}}_2 z + \bar{\bar{\phi}}_3 t_1 + \bar{\bar{\phi}}_4 t_2)$$

$$\rho_{\alpha\beta} = -\phi_\alpha\phi_\beta L_2 + \frac{1}{2}[\phi_\alpha\rho_\beta + \phi_\beta\rho_\alpha],$$

$$\sigma_{\alpha\beta} = -\bar{\rho}_{\alpha\beta} + \frac{1}{4}[\phi_\alpha\phi_\beta L_1 - 2L_2(\phi_\alpha\rho_\beta + \phi_\beta\rho_\alpha) + 2\rho_\alpha\rho_\beta]$$

It is to be noted that the format of the mathematical expressions derived by Takeno (1961) in V_4 is retained even in five dimensional space-time V_5 with three time axes.In the present paper, we confine ourselves to the same space-time V_5 having three time Axes but relax the conditions (1.2), (1.3) and (1.5) with assuming.

$$Z = Z(y, z, t_1, t_2, t_3) \quad Z,_1 \neq 0, \; Z,_2 \neq 0, \; Z,_3 \neq 0, \; Z,_4 \neq 0, \; Z,_5 \neq 0 \tag{1.7}$$

We get some interesting result in GR theory

2. Solutions of field equations

From the equations (1.3) and (1.7)

$$g^{11}\phi_1^2 + 2g^{12}\phi_1\phi_2 + 2g^{13}\phi_1\phi_3 + 2g^{14}\phi_1\phi_4 + 2g^{15}\phi_1 + g^{22}\phi_2^2 + 2g^{23}\phi_2\phi_3 + 2g^{24}\phi_2\phi_4$$

$$+ 2g^{25}\phi_2 + g^{33}\phi_3^2 + 2g^{34}\phi_3\phi_4 + 2g^{35}\phi_3 + g^{44}\phi_4^2 + 2g^{45}\phi_4 + g^{55} = 0 \quad (2.1)$$

Where $\phi_1 = \frac{Z,_1}{Z,_5}$, $\phi_2 = \frac{Z,_2}{Z,_5}$, $\phi_3 = \frac{Z,_3}{Z,_5}$, $\phi_4 = \frac{Z,_4}{Z,_5}$ (2.2)

which further yield

$$w = \phi_\alpha x^\alpha = \phi_1 y + \phi_2 z + \phi_3 t_1 + \phi_4 t_2 + t_3 \quad (2.3)$$

where *w* is an arbitrary function of *Z*.

Differentiating partially (6.2.9) with respect to *y*, *z*, t_1, t_2 and t_3 respectively, we obtain

$$Z,_1 = \frac{\phi_1}{M},\quad Z,_2 = \frac{\phi_2}{M},\quad Z,_3 = \frac{\phi_3}{M},\quad Z,_4 = \frac{\phi_4}{M},\quad Z,_5 = \frac{1}{M} \quad (2.4)$$

Where $M = \bar{w} - \bar{\phi}_\alpha x^\alpha = \bar{w} - (\bar{\phi}_1 y + \bar{\phi}_2 z + \bar{\phi}_3 t_1 + \bar{\phi}_4 t_2)$. (2.5)

From equation (6.2.11), we obtain

$$M,_1 = \frac{N}{M}\phi_1 - \bar{\phi}_1,\quad M,_2 = \frac{N}{M}\phi_2 - \bar{\phi}_2,\quad M,_3 = \frac{N}{M}\phi_3 - \bar{\phi}_3,$$

$$M,_4 = \frac{N}{M}\phi_4 - \bar{\phi}_4,\quad M,_5 = \frac{N}{M} \quad (2.6)$$

Where $N = \bar{\bar{w}} - \bar{\bar{\phi}}_\alpha x^\alpha = \bar{\bar{w}} - (\bar{\bar{\phi}}_1 y + \bar{\bar{\phi}}_2 z + \bar{\bar{\phi}}_3 t_1 + \bar{\bar{\phi}}_4 t_2)$ (2.7)

and a bar (-) over a letter denotes the derivative with respect to *Z*.

It is interesting that all the results are in the format of Takeno (1961).

The Christoffel's symbols of second kind assume the values as follows:

$$2M\Gamma_{11}^i = 2\phi_1 g^{ij}\bar{g}_{1j} - \bar{g}_{11} w^i$$

$$2M\Gamma^i_{22} = 2\phi_2 g^{ij}\overline{g}_{2j} - \overline{g}_{22}w^i,$$

$$2M\Gamma^i_{33} = 2\phi_3 g^{ij}\overline{g}_{3j} - \overline{g}_{33}w^i,$$

$$2M\Gamma^i_{44} = 2\phi_4 g^{ij}\overline{g}_{4j} - \overline{g}_{44}w^i,$$

$$2M\Gamma^i_{55} = 2g^{ij}\overline{g}_{5j} - \overline{g}_{55}w^i,$$

$$2M\Gamma^i_{12} = g^{ij}(\phi_2\overline{g}_{1j} + \phi_1\overline{g}_{2j}) - \overline{g}_{12}w^i,$$

$$2M\Gamma^i_{13} = g^{ij}(\phi_3\overline{g}_{1j} + \phi_1\overline{g}_{3j}) - \overline{g}_{13}w^i,$$

$$2M\Gamma^i_{14} = g^{ij}(\phi_1\overline{g}_{4j} + \phi_4\overline{g}_{1j}) - \overline{g}_{14}w^i,$$

$$2M\Gamma^i_{15} = g^{ij}(\overline{g}_{1j} + \phi_1\overline{g}_{5j}) - \overline{g}_{15}w^i,$$

$$2M\Gamma^i_{23} = g^{ij}(\phi_3\overline{g}_{2j} + \phi_2\overline{g}_{3j}) - \overline{g}_{23}w^i,$$

$$2M\Gamma^i_{24} = g^{ij}(\phi_4\overline{g}_{2j} + \phi_2\overline{g}_{4j}) - \overline{g}_{24}w^i,$$

$$2M\Gamma^i_{25} = g^{ij}(\overline{g}_{2j} + \phi_2\overline{g}_{5j}) - \overline{g}_{25}w^i,$$

$$2M\Gamma^i_{34} = g^{ij}(\phi_3\overline{g}_{4j} + \phi_4\overline{g}_{3j}) - \overline{g}_{34}w^i,$$

$$2M\Gamma^i_{35} = g^{ij}(\overline{g}_{3j} + \phi_3\overline{g}_{5j}) - \overline{g}_{35}w^i,$$

$$2M\Gamma^i_{45} = g^{ij}(\overline{g}_{4j} + \phi_4\overline{g}_{5j}) - \overline{g}_{45}w^i \qquad (2.8)$$

where $w^i = \phi_\alpha g^{\alpha i} = \phi_1 g^{1i} + \phi_2 g^{2i} + \phi_3 g^{3i} + \phi_4 g^{4i} + g^{5i}$. (2.9)

Noting w^i, the equation (6.1) reduces to

$$\phi_\alpha w^\alpha = \phi_1 w^1 + \phi_2 w^2 + \phi_3 w^3 + \phi_4 w^4 + w^5 = 0. \qquad (2.10)$$

The field equations $R_{ij} = 0$ then imply

$$N\rho_{\alpha\beta} + M\sigma_{\alpha\beta} = 0, \quad \alpha, \beta = 1,2.3,4,5. \tag{2.11}$$

Substituting the values of M and N , equation (2.11) reduces to

$$\begin{aligned}
&\overline{\overline{w}}\rho_{\alpha\beta} + \overline{w}\sigma_{\alpha\beta} = 0, \\
&\overline{\overline{\phi}}_1\rho_{\alpha\beta} + \overline{\phi}_1\sigma_{\alpha\beta} = 0, \\
&\overline{\overline{\phi}}_2\rho_{\alpha\beta} + \overline{\phi}_2\sigma_{\alpha\beta} = 0, \\
&\overline{\overline{\phi}}_3\rho_{\alpha\beta} + \overline{\phi}_3\sigma_{\alpha\beta} = 0, \\
&\overline{\overline{\phi}}_4\rho_{\alpha\beta} + \overline{\phi}_4\sigma_{\alpha\beta} = 0
\end{aligned} \tag{2.12}$$

which are again in the format of Takeno (1961).

where $\rho_{\alpha\beta} = -\phi_\alpha\phi_\beta L_2 + \frac{1}{2}[\phi_\alpha\rho_\beta + \phi_\beta\rho_\alpha],$

$$\sigma_{\alpha\beta} = -\overline{\rho}_{\alpha\beta} + \frac{1}{4}[\phi_\alpha\phi_\beta L_1 - 2L_2(\phi_\alpha\rho_\beta + \phi_\beta\rho_\alpha) + 2\rho_\alpha\rho_\beta]$$

i.e. $\sigma_{11} = -\overline{\rho}_{11} + \frac{1}{4}[\phi_1^2 L_1 - 4L_2\phi_1\rho_1 + 2\rho_1^2],$

$$\sigma_{22} = -\overline{\rho}_{22} + \frac{1}{4}[\phi_2^2 L_1 - 4L_2\phi_2\rho_2 + 2\rho_2^2],$$

$$\sigma_{33} = -\overline{\rho}_{33} + \frac{1}{4}[\phi_3^2 L_1 - 4L_2\phi_3\rho_3 + 2\rho_3^2],$$

$$\sigma_{44} = -\overline{\rho}_{44} + \frac{1}{4}[\phi_4^2 L_1 - 4L_2\phi_4\rho_4 + 2\rho_4^2],$$

$$\sigma_{55} = -\overline{\rho}_{55} + \frac{1}{4}[L_1 - 4L_2\rho_5 + 2\rho_5^2],$$

$$\sigma_{12} = -\overline{\rho}_{12} + \frac{1}{4}[\phi_1\phi_2 L_1 - 2L_2(\phi_1\rho_2 + \phi_2\rho_1) + 2\rho_1\rho_2],$$

$$\sigma_{13} = -\bar{\rho}_{13} + \frac{1}{4}[\phi_1\phi_3 L_1 - 2L_2(\phi_1\rho_3 + \phi_3\rho_1) + 2\rho_1\rho_3],$$

$$\sigma_{14} = -\bar{\rho}_{14} + \frac{1}{4}[\phi_1\phi_4 L_1 - 2L_2(\phi_1\rho_4 + \phi_4\rho_1) + 2\rho_1\rho_4],$$

$$\sigma_{15} = -\bar{\rho}_{15} + \frac{1}{4}[\phi_1 L_1 - 2L_2(\phi_1\rho_5 + \rho_1) + 2\rho_1\rho_5],$$

$$\sigma_{23} = -\bar{\rho}_{23} + \frac{1}{4}[\phi_2\phi_3 L_1 - 2L_2(\phi_2\rho_3 + \phi_3\rho_2) + 2\rho_2\rho_3],$$

$$\sigma_{24} = -\bar{\rho}_{24} + \frac{1}{4}[\phi_2\phi_4 L_1 - 2L_2(\phi_2\rho_4 + \phi_4\rho_2) + 2\rho_2\rho_4],$$

$$\sigma_{25} = -\bar{\rho}_{25} + \frac{1}{4}[\phi_2 L_1 - 2L_2(\phi_2\rho_5 + \rho_2) + 2\rho_2\rho_5],$$

$$\sigma_{34} = -\bar{\rho}_{34} + \frac{1}{4}[\phi_3\phi_4 L_1 - 2L_2(\phi_3\rho_4 + \phi_4\rho_3) + 2\rho_3\rho_4],$$

$$\sigma_{35} = -\bar{\rho}_{35} + \frac{1}{4}[\phi_3 L_1 - 2L_2(\phi_3\rho_5 + \rho_3) + 2\rho_3\rho_5],$$

$$\sigma_{45} = -\bar{\rho}_{45} + \frac{1}{4}[\phi_4 L_1 - 2L_2(\phi_4\rho_5 + \rho_4) + 2\rho_4\rho_5],$$

$$\rho_{11} = -\phi_1^2 L_2 + \phi_1\rho_1,$$
$$\rho_{22} = -\phi_2^2 L_2 + \phi_2\rho_2,$$

$$\rho_{33} = -\phi_3^2 L_2 + \phi_3\rho_3,$$
$$\rho_{44} = -\phi_4^2 L_2 + \phi_4\rho_4,$$

$$\rho_{55} = -L_2 + \rho_5,$$

$$\rho_{12} = -\phi_1\phi_2 L_2 + \frac{1}{2}[\phi_1\rho_2 + \phi_2\rho_1],$$
$$\rho_{13} = -\phi_1\phi_3 L_2 + \frac{1}{2}[\phi_1\rho_3 + \phi_3\rho_1],$$

$$\rho_{14} = -\phi_1\phi_4 L_2 + \frac{1}{2}[\phi_1\rho_4 + \phi_4\rho_1],$$

$$\rho_{15} = -\phi_1 L_2 + \frac{1}{2}[\phi_1\rho_5 + \rho_1],$$

$$\rho_{23} = -\phi_2\phi_3 L_2 + \frac{1}{2}[\phi_2\rho_3 + \phi_3\rho_2],$$

$$\rho_{24} = -\phi_2\phi_4 L_2 + \frac{1}{2}[\phi_2\rho_4 + \phi_4\rho_2],$$

$$\rho_{25} = -\phi_2 L_2 + \frac{1}{2}[\phi_2\rho_5 + \rho_2],$$

$$\rho_{34} = -\phi_3\phi_4 L_2 + \frac{1}{2}[\phi_3\rho_4 + \phi_4\rho_3],$$

$$\rho_{35} = -\phi_3 L_2 + \frac{1}{2}[\phi_3\rho_5 + \rho_3],$$

$$\rho_{45} = -\phi_4 L_2 + \frac{1}{2}[\phi_4\rho_5 + \rho_4]$$

with $\rho_i = \bar{g}_{ij} w^j$, $L_2 = \overline{\log \sqrt{g}}$, $L_1 = g^{ij} g^{kl} \bar{g}_{ik} \bar{g}_{jl}$.

It is to be noted that all the results follow Takeno's (1961) pattern.

CONCLUSION 1

We conclude that the plane wave solutions exist in higher five dimensional space-times V_5 *having three time axes and are given by* g_{ij} *satisfying equations* (1.2), (1.5), (2.10) and (2.12).

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