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STUDIES ON MINKOWSKI SPACE FOR SOLUTION OF EINSTEIN EQUATIONS

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ABSTRACT

The problem of the stability of Minkowski space time has been discussed, and then we have given the simplest solution of the Einstein equations for space time.

INTRODUCTION

The Minkowski space time is the simplest solution of the Einstein equations.[1-3] I.e.

$$(\mathbb{R}^4, \eta) \quad , \qquad \eta_{\mu\nu} = diag\,(-1,1,1,1) \tag{1}$$

in rectangular coordinates.

The function x^0 corresponding to any rectangular coordinate system is a canonical maximal time function. The level sets Σ_t (x^0= t) are maximal spacelike hypersurfaces. Here they happen to be totally geodesic (not only is trk = 0 but k_{ij} = 0 identically). They are also globally parallel.[4-5] (The lapse function is Φ = 1 identically.) Cauchy problem with initial data on a complete maximal hypersurface.

Let (H_0, g_0, k_0) with $H_0 \approx$ R3.

(H_0, g_0) is asymptotically Euclidean in the strong sense. I.e. there exists a coordinate system in the neighbourhood of infinity in which the metric coefficients obey

$$\bar{g}_{0_{ij}} = (1 + \frac{2M}{r})\,\delta_{ij} + O_2\,(r^{-1-\epsilon}) \;\;,\;\; \epsilon > 0 \tag{2}$$

$$tr\; k_0 = 0 \tag{3}$$

$$k_0 = O_1\,(r^{-2-\epsilon}) \tag{4}$$

This implies that the total linear momentum vanishes. $P^i = 0$.
The initial data set is to satisfy the constraint equations.

$$\bar{\nabla}^i k_{ij} = 0 \quad \text{Codazzi} , \tag{5}$$

$$\bar{R} = |k|^2 \quad \text{Gauss} \tag{6}$$

The hypotheses are that we are given such an initial data set. Now the problem to solve is the following.

Problem. Supplement these hypotheses by a smallness condition and show that we can then construct a geodesically complete solution of the Einstein equations, tending to the Minkowski spacetime along any geodesic.

FIELD THEORIES IN A GIVEN SPACETIME

We write for the Lagrangian

$$L = L^* \, d_{\mu g},$$

where L* is a scalar function, constructed out of the fields, their exterior or more generally covariant derivatives, the metric and connection coefficients. We shall give now 3 examples.

Scalar field Φ.

Let

$$\begin{aligned} \sigma &= g^{\mu\nu} \, \partial_\mu \, \Phi \, \partial_\nu \, \Phi \\ L^* &= L^* \, (\sigma) \; . \end{aligned}$$

Only exterior derivatives are involved. So, the Lagrangian does not depend on the connection coefficients.

Electromagnetic field $F_{\mu\nu}$.

$$\begin{aligned} F &= dA \quad (F_{\mu\nu} = \partial_\mu \, A_\nu - \partial_\nu \, A_\mu) \; , \\ \alpha &= F^{\mu\nu} \, F_{\mu\nu} \;\; ; \;\; F^{\mu\nu} = g^{\mu\kappa} \, g^{\nu\lambda} \, F_{\kappa\lambda} \; , \\ \beta &= F^{\mu\nu} \, {}^*F_{\mu\nu} \;\; ; \;\; {}^*F_{\mu\nu} = \frac{1}{2} \, F^{\kappa\lambda} \, \epsilon_{\kappa\lambda\mu\nu} \; , \\ L^* &= L^* \, (\alpha, \beta) \; . \end{aligned}$$

Here we have

Also here, only exterior derivatives are involved and the Lagrangian does not depend on the connection coefficients.

Let (M,g) be a compact Riemannian manifold. Find a vectorfield U generating an appropriate isometry of (M,g).

The solution is the following. Consider

$$\pi = LUg ,$$

the deformation tensor. It measures the deviation from an isometry. Minimize

$$\int_M |\pi|^2 \, d\mu_g$$

under the constraint

$$\int_M |U|^2 \, d\mu_g \;=\; 1$$

The Euler-Lagrange equation is the eigenvalue problem.

$$\begin{aligned} div\ \pi \;+\; \lambda\, U \;&=\; 0\,, \qquad \text{that is} \\ \nabla_j\, \pi^{ij} \;+\; \lambda\, U^i \;&=\; 0\,, \end{aligned}$$

where λ is the Lagrange multiplier or eigenvalue. The analogous problem on a spacetime manifold reads as.

$$\begin{aligned} L^*\, d\mu_g \;&=\; \pi^{\mu\nu}\, \pi_{\mu\nu}\, d\mu_g\,, \\ \pi^{\mu\nu} \;&=\; g^{\mu\nu}\, g^{\nu\lambda}\, \pi_{\kappa\lambda}\,, \\ \pi_{\mu\nu} \;&=\; \mathcal{L}_U g_{\mu\nu} \;=\; \nabla_\mu\, U_\nu \;+\; \nabla_\nu\, U_\mu \\ U_\mu \;&=\; g_{\mu\nu}\, U^\nu\,. \end{aligned}$$

Here, L* depends on the covariant derivatives of U, therefore the connection coefficients corresponding to g.

CONCLUSION

This problem has a direct application in General Relativity. The problem in General Relativity is that of preservation of symmetry. Namely, to show that if the initial conditions possess a continuous isometry group then the solution also possesses the same isometry group. The difficulty in General Relativity, which is absent for the analogous problem in the case of a theory in a given space time, is to extend the action of the group from the initial hyper surface to the space time manifold.

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